Physics 721, Fall 2006 Josh Erlich

Final Exam. Due 5:00pm Tuesday, December 12, 2006.

You may consult any notes you have taken, any notes I have given you, and Peskin & Schroeder's textbook. That is all you may consult, out of fairness to your classmates. You should work alone. *Hint*: You will find the gamma matrix trace and contraction identities in Chapter 5.1 of Peskin & Schroeder.

If you have any questions, feel free to stop by my office, email me at erlich@physics.wm.edu, or phone me on my cell phone at (757)272-2697. I will post answers to any questions I receive on the course website at: http://physics.wm.edu/∼erlich/721F06/

There are two problems on the exam. You are expected to answer all parts of both problems. You should not spend more than six hours on the exam.

1. $\mu \to e\gamma$ (50 points)

In this problem you will study the process $\mu \to e\gamma$, a process which does not occur in the Standard Model in the absence of neutrino masses.

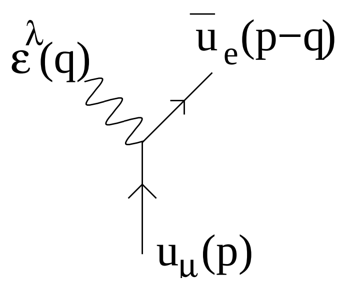
a) (5 points) The muon is identical to the electron except for its mass. What is the Lagrangian density for electrons and muons coupled to electromagnetism?

b) (10 points) What are the conserved charges due to the non-spacetime symmetries in this theory, and their physical interpretation?

c) (5 points) Explain, based on symmetries, why the process $\mu \to e\gamma$ is forbidden in this theory.

For the rest of this problem we will approximate the electrons as being massless.

d) (20 points) Neutrino masses can violate the symmetries which prohibit leptonic flavor-changing processes like $\mu \to e\gamma$. It can be argued based on Lorentz symmetry and gauge invariance that in the massless electron limit, the effective $\mu \to e\gamma$ vertex must have the form:



$$iM(\mu \to e\gamma) = A\,\varepsilon^\lambda(q)\,\overline{u}_e(p-q)\;(iq^\nu\,[\gamma_\lambda, \gamma_\nu]\,(1+\gamma_5))\;u_\mu(p),$$

where A is some constant, u_e and u_μ are the Dirac spinors describing the electron and muon, and ε^λ is the photon polarization vector.

What is the decay rate of unpolarized muons into electrons and photons in the muon rest frame?

e) (5 points) What is the probability that the Dirac spinor describing the final state electron is of left handed chirality, *i.e.* $\gamma_5 u_e = -u_e$?

f) (5 points) In part (d) you calculated the decay rate of unpolarized muons. Instead, now imagine the initial state muons have right handed chirality, *i.e.* $\gamma_5 u_\mu = u_\mu$. What is the decay rate of these muons into electrons and photons?

Note: Since in part (f) the chirality eigenstates are not Hamiltonian eigenstates, part (f) is not really a physical problem. I just want you to pretend that it is, and calculate the tree level decay rate of right-handed muons starting from the Feynman diagram in part (d).

2. *Nucleon-Meson Interactions* (50 points)

In this problem you will consider scattering of spin-1/2 nucleons, ψ, due to the interaction with a real scalar field, ϕ. This theory is similar to Yukawa's model for nucleon-meson interactions, except that the pions are pseudoscalar particles while here we are treating them as scalars.

The Lagrangian density of this model is,

$$\mathcal{L} = \overline{\psi}\,(i\partial\!\!\!/ - M)\,\psi + \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - g\,\overline{\psi}\phi\psi.$$

a) (20 points) What is the differential cross section $d\sigma/d\Omega$ in the center-of-mass frame for unpolarized nucleon-nucleon scattering, summed over final state nucleon spins, to the lowest nontrivial order in g?

You should calculate all gamma matrix traces, and your final expression should be in terms of kinematical factors. You should evaluate all kinematical factors in terms of the total energy, particle masses, and scattering angle, but you do not have to simplify the final result.

b) (10 points) By comparing with the Born approximation in nonrelativistic quantum mechanics, what are the nonrelativistic potential and exchange potential due to the exchange of ϕ particles?

c) (5 points) Do nucleons attract or repel other nucleons in this theory?

d) (10 points) Repeat part (b) for nucleon-antinucleon scattering.

e) (5 points) Do nucleons and antinucleons attract or repel in this theory?