Physics 721, Fall 2005 Josh Erlich
Problem Set 8: Vector fields
Due Thursday, November 17.

1. *Massive electrodynamics*

The most general Lagrangian density for the vector field including terms quadratic in the field with at most two derivatives is (up to the addition of total derivatives):

$$\mathcal{L} = -\frac{1}{2}\,\partial_\mu A_\nu\,\partial^\mu A^\nu + b\,\partial_\mu A_\nu \partial^\nu A^\mu + c\,A_\mu A^\mu.$$

(a) What are the Euler-Lagrange equations for this theory?

(b) Assume a plane wave solution of the form $A^\mu(x) = \varepsilon^\mu(k)e^{-ik\cdot x}$. What are the Euler-Lagrange equations in terms of ε^μ and k?

(A^μ is real, but as usual we are describing two solutions at once – the real and imaginary parts of the plane wave. We can do this because the Euler-Lagrange equations are linear in this theory.)

(c) **Longitudinal mode:** Assume $\varepsilon^\mu(k) \propto k^\mu$. What are the Euler-Lagrange equations for this *ansatz* in terms of ε^μ? Define $k_\mu k^\mu \equiv m_L^2$. What is the longitudinal mass m_L in terms of the parameters b and c in the Lagrangian?

(d) **Transverse modes:** Repeat part (c) assuming that $\varepsilon_\mu k^\mu = 0$. This time define $k_\mu k^\mu \equiv m_T^2$. What is m_T?

(e) The longitudinal mode will not propagate if $m_L \to \infty$. What choice of b accomplishes this? Make that choice and rewrite $\mathcal{L}$ in terms of A^μ, m_T, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This is the Proca Lagrangian of massive electrodynamics. You should recover Maxwell's theory if you take $m_T \to 0$.

2. *Compton scattering*

Calculate the scattering amplitude at $\mathcal{O}(e^2)$ for Compton scattering in QED, $e^- + \gamma \to e^- + \gamma$. Draw the Feynman diagrams and label the appropriate quantum numbers on all external lines.