Physics 721, Fall 2005 Josh Erlich
Problem Set 6: Wick's theorem
Due Tuesday, November 1.

1. *Wick's theorem for fermions*

Prove Wick's theorem for fermions: For any free fermionic fields $A_i(x_i)$,

$$\begin{aligned} T[A_1(x_1)\,A_2(x_2)\cdots A_n(x_n)] \;&=\; :A_1(x_1)\cdots A_n(x_n): \\ &\quad + \text{all normal} - \text{ordered contractions.} \end{aligned}$$

The contractions are defined as in the bosonic case:

$$A(x)B(y) = T[A(x)B(y)] - :A(x)B(y):$$

The sign rules for exchange of fermionic fields are as follows:

$T[A(x)B(y)C(z)] = -T[A(x)C(z)B(y)]$, *i.e.* a minus sign for each exchange of neighboring fermion fields under the time-ordering symbol.

$:A(x)B(y)C(z): = -:A(x)C(z)B(y):$, *i.e.* a minus sign for each exchange of neighboring fermion fields under the normal-ordering symbol.

$:A(x)B(y)C(z)D(w): = -A(x)C(z):B(y)D(w):$, *i.e.* a minus sign for each exchange of neighboring fermion fields required to make contracted fields neighbors.

2. *Nuclear processes*

The Lagrangian density for the meson-nucleon theory is,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\mu^2}{2}\phi^2 + \overline{\psi}(i\partial\!\!\!/ - m)\psi - g\,\overline{\psi}\psi\phi.$$

Enumerate the leading-order Wick diagrams which contribute to all 2-body scattering processes in this theory. For each diagram, write its contribution to the scattering matrix and list the various processes that the diagram contributes to (*e.g.* N+N $\to$ N+N). Draw only a single diagram corresponding to each pair of contracted fields, but label the vertices and draw a diagram for each *independent* ordering of the vertices.