1. **Wick’s theorem for fermions**

Prove Wick’s theorem for fermions: For any free fermionic fields $A_i(x_i)$,

$$T[A_1(x_1)A_2(x_2)\cdots A_n(x_n)] = :A_1(x_1)\cdots A_n(x_n):$$

+ all normal-ordered contractions.

The contractions are defined as in the bosonic case:

$$A(x)B(y) = T[A(x)B(y)] - :A(x)B(y):$$

The sign rules for exchange of fermionic fields are as follows:

$$T[A(x)B(y)C(z)] = -T[A(x)C(z)B(y)], \text{ i.e. a minus sign for each exchange of neighboring fermion fields under the time-ordering symbol.}$$

$$:A(x)B(y)C(z): = -:A(x)C(z)B(y):, \text{ i.e. a minus sign for each exchange of neighboring fermion fields under the normal-ordering symbol.}$$

$$:A(x)B(y)C(z)D(w): = -A(x)C(z):B(y)D(w):, \text{ i.e. a minus sign for each exchange of neighboring fermion fields required to make contracted fields neighbors.}$$

2. **Nuclear processes**

The Lagrangian density for the meson-nucleon theory is,

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\mu^2}{2} \phi^2 + \bar{\psi} (i\gamma^\mu - m) \psi - g \bar{\psi} \gamma^5 \phi.$$ 

Enumerate the leading-order Wick diagrams which contribute to all 2-body scattering processes in this theory. For each diagram, write its contribution to the scattering matrix and list the various processes that the diagram contributes to (e.g. $N+N \to N+N$). Draw only a single diagram corresponding to each pair of contracted fields, but label the vertices and draw a diagram for each independent ordering of the vertices.