Physics 721, Fall 2005JoProblem Set 5: Review of Dirac Fields, SymmetriesDue Thursday, October 13.

Reminder: The midterm exam will be available in my office and online Friday, October 14 at 3pm, and will be due at the beginning of class the following Tuesday.

1. Gamma matrices

Show that:

$$\gamma^5 \gamma^\mu \gamma^5 = -\gamma^\mu$$
$$\operatorname{Tr} \phi = 0$$
$$\operatorname{Tr} \phi \phi = 4a \cdot b$$

Similarly, compute Tr $\not a \not b \not c$, Tr $\not a \not b \not c \not d$, Tr $\not a \gamma^5$, Tr $\not a \not b \gamma^5$, Tr $\not a \not b \not c \gamma^5$, and Tr $\not a \not b \not c \not d \gamma^5$.

The last of these will involve the constant antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$. Use only the anticommutation relations of the 4×4 gamma matrices. Do not use an explicit representation of the matrices. Recall that $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

2. Electric charge of the electron

The Dirac Lagrangian density is,

$$\mathcal{L} = \overline{\psi}(i\partial \!\!\!/ - m)\psi.$$

The transformation $\psi \to e^{i\theta} \psi$, $\overline{\psi} \to e^{-i\theta} \overline{\psi}$ is a symmetry of the theory.

In terms of an integral over the fields, what is the associated 4-vector current? What is the charge?

3. Chiral symmetry

Consider the Dirac Lagrangian again. Does the transformation $\psi \rightarrow e^{i\theta\gamma^5}\psi, \psi^{\dagger} \rightarrow \psi^{\dagger}e^{-i\theta\gamma^5}$ leave the Lagrangian invariant?

Can the Lagrangian be made invariant by setting either of the terms in the Lagrangian to zero? If so, then what is the associated 4-vector current? What is the charge?

The up and down quarks are light compared to the masses of the pions and nucleons made in accelerators, so it is a good approximation to treat them as massless. The subject of making predictions about observed particles by treating the quark masses as perturbations and using the approximately conserved current which you just constructed is called **chiral perturbation theory**.