Physics 721, Fall 2005 Josh Erlich

Problem Set 4: More Quantized Scalar Fields

Due Tuesday, October 4.

1. *4-momentum as generator of space-time translations*

Consider a free real scalar field. Using the commutation relations between the free field Hamiltonian and the raising and lowering operators, show that:

$$e^{iHt}\, a_{\mathbf{k}}\, e^{-iHt} = a_{\mathbf{k}} e^{-i\omega_{\mathbf{k}} t}$$
$$e^{iHt}\, a_{\mathbf{k}}^\dagger\, e^{-iHt} = a_{\mathbf{k}}^\dagger\, e^{i\omega_{\mathbf{k}} t}$$
$$e^{-i\mathbf{P}\cdot\mathbf{x}}\, a_{\mathbf{k}}\, e^{i\mathbf{P}\cdot\mathbf{x}} = a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$$
$$e^{-i\mathbf{P}\cdot\mathbf{x}}\, a_{\mathbf{k}}^\dagger\, e^{i\mathbf{P}\cdot\mathbf{x}} = a_{\mathbf{k}}^\dagger\, e^{-i\mathbf{k}\cdot\mathbf{x}}$$

where $\mathbf{P} = \int \frac{d^3k}{(2\pi)^3}\, \mathbf{k}\, a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$ and $H = \int \frac{d^3k}{(2\pi)^3}\, \omega_{\mathbf{k}}\, a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$

Using this, show that,

$$\phi(x) = e^{i(Ht-\mathbf{P}\cdot\mathbf{x})}\, \phi(0)\, e^{-i(Ht-\mathbf{P}\cdot\mathbf{x})}$$

Hence, the 4-momentum operator $(H, \mathbf{P})$ generates translations of the field $\phi(x)$ in space-time. This is a specific example of a general phenomenon: the conserved charge under some symmetry generates the corresponding symmetry transformation on the fields.

2. *Green's functions for complex scalar fields*

Compute $\langle 0|\phi^\dagger(x)\phi(y)|0\rangle$, $\langle 0|\phi(x)\phi(y)|0\rangle$, $\langle 0|T(\phi(x)\phi(y))|0\rangle$, and $\langle 0|T(\phi^\dagger(x)\phi(y))|0\rangle$. If your results are nonvanishing, you should leave them in the form of an integral over a single 4-momentum d^4k.

With or without doing a calculation, argue from what you already know about the real scalar field that observables built out of the complex scalar field commute at spacelike separation.