Physics 690/482-02, Spring 2005 Josh Erlich

Midterm Quiz: Tensor vs Scalar Gravity in Warped Extra Dimensions

In class we derived the gravitational potential in the Randall-Sundrum model by studying Einstein's equations for transverse-traceless metric fluctuations. Here you will compare the equations of motion for the metric fluctuations and for a scalar field in the same gravitational background.

The Randall-Sundrum metric is of the form,

$$ds^2 = e^{-A(z)}(dx_4^2 + dz^2) \ .$$

The generally covariant action for a massless scalar field is,

$$S = \int d^4x \, dz \sqrt{|g|} \, \frac{1}{2} \, g^{MN} \, \partial_M \phi \partial_N \phi \ ,$$

from which the scalar field equation of motion follows:

$$\frac{1}{\sqrt{|g|}} \, \partial_M \left(\sqrt{|g|} \, g^{MN} \, \partial_N \phi \right) = 0 \ .$$

1. What is the equation of motion for the scalar field in the warped geometry in terms of $A(z)$ and $\phi(x,z)$?

2. Separate variables, *i.e.* write the scalar field as $\phi(x,z) = \tilde{\phi}(x) \, \psi(z)$. What are the equations of motion for $\tilde{\phi}(x)$ and $\psi(z)$?

Define $\Box_4 \tilde{\phi}(x) = m^2 \, \tilde{\phi}(x)$. How does the mass m appear in the equation for $\psi(z)$?

3. Define $\psi(z) = e^{3A(z)/4} \, \tilde{\psi}(z)$. What is the Schrödinger-like equation for $\tilde{\psi}(z)$?

Compare this with the Schrödinger equation derived in class for the tensor gravity wavefunction.

April Fools. This is Problem Set 9, and it will be due Friday, April 8.