Physics 690/482-02, Spring 2005 Josh Erlich

Problem Set 8: Warped Extra Dimensions - The Background

1. *Fat Branes*

Assume a 5D metric of the form $ds^2 = e^{-A(y)}\,dx^\mu\,dx^\nu\,\eta_{\mu\nu} + dy^2$. (As usual, Greek indices run from 0 to 3, and capital Latin indices will run from 0 to 4.)

- Calculate the Christoffel symbols $\Gamma^y_{\mu\nu}$ and $\Gamma^\mu_{\nu y}$.
- Calculate the nonvanishing components of the Ricci tensor, $R_{\mu\nu}$ and R_{yy}. *Hint:* The Ricci tensor can be written as,

$$R_{MN} = \partial_P\,\Gamma^P_{MN} - \partial_M\,\Gamma^P_{PN} + \Gamma^Q_{MN}\Gamma^P_{QP} - \Gamma^Q_{PN}\Gamma^P_{QM}\ .$$

- Calculate the curvature scalar, $R = g^{MN}\,R_{MN}$.
- Derive the form of the stress tensor T_{MN} which is consistent with the above metric and Einstein's equations, $R_{MN} - \frac{1}{2}\,g_{MN}\,R = 8\pi G_N\,T_{MN}$.
- Write the stress tensor in the form,

$$T_M^{\ N} = \begin{pmatrix} \Lambda(y)+V(y) & & & & \\ & \Lambda(y)+V(y) & & & \\ & & \Lambda(y)+V(y) & & \\ & & & \Lambda(y)+V(y) & \\ & & & & \Lambda(y) \end{pmatrix}.$$

  What are $\Lambda(y)$ and $V(y)$? (*Hint:* Be careful. Remember, $T_M^{\ N} = T_{MP}\,g^{PN}$.)
- If $A(y) = k|y|$ for some constant k, then derive the relation between the cosmological constant Λ and the brane tension V discussed in class. (*Hint:* Recall that $8\pi G_N = 1/(2M_{Pl}^3)$ in 4+1 dimensions.)
- If $A(y)$ is a smooth function that approximates $k|y|$, then plot (roughly) $\Lambda(y)$ and $V(y)$.

You are welcome to use a symbolic manipulation program like Mathematica to help you, but it shouldn't be necessary.