Physics 690/482-02, Spring 2005 Josh Erlich
Problem Set 7: Large Extra Dimensions

1. *Energies, Masses and Lengths, Oh my!*

In class we have been cavalier about speaking about length scales, energy scales and mass scales interchangeably. In this problem you will derive the relations among these units. In the units most commonly used in textbooks,

$$\begin{aligned}\hbar &\simeq 1.05\times 10^{-34}\ \mathrm{m^2\,kg/s}\\ c &\simeq 3\times 10^{8}\ \mathrm{m/s}\\ G_N &\simeq 6.7\times 10^{-11}\mathrm{m^3/kg\,s^2}\end{aligned}$$

$$\begin{aligned}1\ \mathrm{eV} &\simeq 1.6\times 10^{-19}\ \mathrm{Joules}\\ 1\ \mathrm{GeV} &= 10^{9}\ \mathrm{eV}\\ 1\ \mathrm{Joule} &= 1\ \mathrm{kg\,m^2/s^2}\\ 1\ \mathrm{fermi} &= 10^{-15}\ \mathrm{m}\end{aligned}$$

a. The mass of a proton is around .9 GeV/c^2. Express the mass in kilograms. Using $\hbar$ and c derive a length scale from the proton mass. This length scale is $\lambda_p/(2\pi)$, where λ_p is the Compton wavelength of the proton.

b. Express $8\pi G_N$ as a mass to some power, times factors of $\hbar$ and c. The mass scale is called the **Planck mass.** Using $\hbar$ and c, derive a length scale from the Planck mass. This is called the **Planck length**.

c. A black hole forms when a mass M is squeezed into a region of size $r_S = 2G_N M/c^2$. The **Schwarzschild length** r_S is the size of the horizon of the black hole. How many times bigger is the proton's Compton wavelength than the Schwarzschild length for a black hole of the same mass as the proton?

How big would a black hole be if its horizon size was the same as its Compton wavelength? What would be its mass?

d. The weak force is mediated by heavy particles, the W bosons ($m_W \simeq$80

GeV/c^2) and the Z boson ($m_Z \simeq$91 GeV/c^2). The force that is mediated by these gauge bosons is supressed by the exponential $e^{-m\,|\mathbf{x}-\mathbf{x}'|}$ in the Yukawa potential. For what separation $|\mathbf{x}-\mathbf{x}'|$ is this exponential factor for the W boson equal to 1/100?

2. *Change of basis for wavefunctions*

Consider a braneworld with one large extra dimension of size $2\pi R$. The coordinate along the circle is y, the wavefunction along the circle is $\psi_n(y)$, and the brane is at $y = 0$. Recall the expression we derived for the gravitational potential on the brane:

$$V(|\mathbf{x}-\mathbf{x}'|) = -4\pi G_N^{(5)} \sum_n |\psi_n(0)|^2 \, \frac{e^{-\frac{n\,|\mathbf{x}-\mathbf{x}'|}{R}}}{4\pi|\mathbf{x}-\mathbf{x}'|} \ .$$

The basis we used for the complete set of wavefunctions satisfying the periodic boundary conditions was,

$$\begin{aligned}
\psi_0(y) &= \frac{1}{\sqrt{2\pi R}} \\
\psi_n^{(1)}(y) &= \frac{\cos(ny/r)}{\sqrt{\pi R}}\ , n > 0 \\
\psi_n^{(2)}(y) &= \frac{\sin(ny/r)}{\sqrt{\pi R}}\ , n > 0.
\end{aligned}$$

Alternatively, we could have chosen another basis of wavefunctions, for example,

$$\begin{aligned}
\psi_0(y) &= \frac{1}{\sqrt{2\pi R}} \\
\psi_n^{(1)}(y) &= \frac{\cos(ny/r+\theta)}{\sqrt{\pi R}}\ , n > 0 \\
\psi_n^{(2)}(y) &= \frac{\sin(ny/r+\theta)}{\sqrt{\pi R}}\ , n > 0
\end{aligned}$$

where θ is an arbitrary phase.

Show that the potential on the brane is the same in both bases.

3. *Final Project*: Choose a topic for your final project, and decide whether you want to give an oral presentation or a written report.