Physics 690/482-02, Spring 2005 Josh Erlich

Problem Set 6: Gauge Invariance and the Low Energy Effective Action

In this problem set you will study how higher dimensional gauge transformations act on the low energy effective theory.

Consider the Lagrangian density for electromagnetism coupled to a massless complex scalar field $\psi(x)$ in $4+1$ dimensions on a circle of radius R:

$$\mathcal{L} = -\frac{1}{4} F_{MN} F^{MN} + |(\partial_M + iq\, A_M)\psi|^2.$$

Call the coordinate in the extra dimension y, and the coordinates in the noncompact dimensions x^μ, $\mu = 0, \ldots, 3$. We use the shorthand x for x^μ.

You showed in Problem Set 5 that the Lagrangian is invariant under the gauge transformation,

$$\begin{aligned} A_M &\rightarrow A_M - \partial_M \alpha(x,y) \;, \quad M = 0, \ldots, 5 \\ \psi &\rightarrow \psi\, e^{iq\,\alpha(x,y)}. \end{aligned}$$

Now you will study gauge transformations of the low energy effective action:

Expand the gauge field and the scalar field in Kaluza-Klein modes:

$$\begin{aligned} A^M(x,y) &= \sum_{n=-\infty}^{\infty} A_n^M(x)\, e^{\frac{iny}{R}} \;, \quad M = 0, \ldots, 4 \\ \psi(x,y) &= \sum_{n=-\infty}^{\infty} \psi_n(x)\, e^{\frac{iny}{R}} \;. \end{aligned}$$

1. Show that a gauge transformation with $\alpha(x,y) = \frac{N}{qR}\, y$ transforms

the zero-modes as follows:

$$
\begin{aligned}
A_0^\mu(x) &\rightarrow A_0^\mu(x), \quad \mu = 0, \dots, 3 \\
A_0^y(x) &\rightarrow A_0^y(x) - \frac{N}{qR} \\
\psi_0(x) &\rightarrow \psi_0(x)\, e^{iN\,y/R}
\end{aligned}
$$

2. Write the low energy effective action by keeping only the zero modes in the Lagrangian and performing the integral over y. How does the gauge transformation above act on the low energy Lagrangian?

3. What is the term containing $\psi_{-N}(x)$ in the Kaluza-Klein expansion of the full Lagrangian? Explain how the Kaluza-Klein modes restore gauge invariance of the action.