

Physics 690/482-02, Spring 2005
Problem Set 3: Compactified extra dimensions

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1. *Colliding superballs in an extra dimension.*

Imagine an elastic collision of two nonrelativistic balls of unequal masses m_1 and m_2 . in a 4+1 dimensional world compactified on a circle of size R . One of the balls has 4-velocity $v \cos \phi \hat{z} + v \sin \phi \hat{\theta}$, while the other has 4-velocity $-(v \cos \phi \hat{z} + v \sin \phi \hat{\theta})$, where $\hat{\theta}$ is a unit vector in the compact direction and \hat{z} is a unit vector in a noncompact direction.

- Using conservation of energy and momentum find the final momenta and velocities of the superballs after the collision.
- Consider an observer who can only measure motion along the noncompact directions. Does this observer find that momentum and energy are conserved? If so, then explain why it was not necessary to modify the mass by a factor depending on the momentum in the compact direction as we have suggested in class. (*Hint: In terms of energy and momentum, when is motion nonrelativistic?*)

2. *Massless fields on a donut.*

Consider a free massless scalar field in 5 + 1 dimensions compactified on a 2-dimensional untwisted torus with radii $R_1 \ll R_2$.

- Including only the lightest Kaluza-Klein mode, what is the 3 + 1 dimensional effective Lagrangian for energies $E \ll 1/R_2$?
- What is the 4+1 dimensional effective Lagrangian for energies $1/R_2 \ll E \ll 1/R_1$?
- Assume a static source $\rho(\mathbf{x})$. What is the potential $V(|\mathbf{x} - \mathbf{x}'|)$? You can leave your answer in terms of Fourier transformed quantities.
- How does the potential vary as a function of $|\mathbf{x} - \mathbf{x}'|$ in the presence of the source for $|\mathbf{x} - \mathbf{x}'| \ll R_1$? For $R_1 \ll |\mathbf{x} - \mathbf{x}'| \ll R_2$? For $|\mathbf{x} - \mathbf{x}'| \gg R_2$? (You can argue for your answer without providing the full calculation.)