Physics 690/482-02, Spring 2005 Josh Erlich
Problem Set 11: The Goldberger-Wise Mechanism
Due Friday, April 29

The RS1 model purports to solve the hierarchy problem by virtue of the fact that there is an exponential suppression of energy scales from the warp factor e^{-2kd} on the TeV brane. However, there is a solution to Einstein's equations for any d, so it would be nice if we had a reason to expect that d is within a few orders of magnitude of $\mathcal{O}(1/k)$. One way to accomplish this is called the **Goldberger-Wise mechanism**, which you will study here. You may wish to consult Goldberger & Wise, hep-ph/9907447.

1. Assume there is a scalar field ϕ which has a bulk mass m and a localized potential on each of the branes, with action,

$$\begin{aligned} S \;=\; & \int d^5x \left[\sqrt{|g_{(5)}|} \left(2M^3 R - \frac{1}{2}(\partial_N \phi)^2 - \frac{m^2}{2}\phi^2 \right) \right. \\ & \left. - \sqrt{|g_h|}\, \lambda_h (\phi^2 - v_h^2)^2\, \delta(y) - \sqrt{|g_v|}\, \lambda_v (\phi^2 - v_v^2)^2\, \delta(y-d) \right] . \end{aligned}$$

The background metric is $ds^2 = e^{-2ky}\, dx^2 + dy^2$ where $0 < y < d$, so that $\sqrt{|g_h|} = 1$ and $\sqrt{|g_v|} = e^{-4kd}$.

Assume that ϕ is a function of y only. *What is the scalar field equation of motion in the background geometry?*

2. Assume a solution of the form $\phi(y) = e^{aky}$ for some constant a. *What are the two independent solutions for a?*

3. The generic solution for ϕ is $\phi = A\, e^{a_+ ky} + B\, e^{a_- ky}$, where a_+ and a_- are the two solutions you found in Problem 2, and A and B are constants.

If λ_v and λ_h are big enough, then by the equations of motion, the scalar field on the two branes will satisfy,

$$\phi(0) \approx v_h, \quad \phi(d) \approx v_v \; .$$

Using these boundary conditions, solve for A and B.

4. Assume that $e^{-kd} \ll 1$. You will show that this assumption is self consistent. *Expand A and B, keeping only the leading exponentials multiplying v_v and v_h.*

5. The **radion** is related to the geometric parameter d which specifies the size of the extra dimension and the position of the TeV brane. In the presence of the scalar field background, a potential is generated for d, which can be seen by carefully solving Einstein's equations.

Another way to see this is to plug the scalar field solution you have found into the action, and integrate over y to determine a potential for d in the effective 4D theory.

Goldberger and Wise did this, assuming that $\epsilon \equiv m^2/4k^2 \ll 1$, and obtained:

$$\begin{aligned} V(d) \;&=\; k\epsilon v_h^2 + 4ke^{-4kd}\left(v_v - v_h\, e^{-\epsilon kd}\right)^2\left(1+\frac{\epsilon}{4}\right) \\ &\quad - k\epsilon v_h\, e^{-(4+\epsilon)kd}\left(2v_v - v_h\, e^{-\epsilon kd}\right) \ . \end{aligned}$$

You can drop all terms in the potential which are higher order in ϵ **except** in the exponentials (because d only appears in the exponentials).

Solve for kd to lowest order in ϵ by minimizing $V(d)$.

One can check that all of the assumptions of smallness of the parameters only requires numbers of order 100 at most, which is much smaller than $M_{Pl}/M_W \approx 10^{16}$.

Hence, you have shown that it is possible to get a large warp factor e^{2kd} and explain the hierarchy between M_{Pl} and M_W, even if all of the parameters in the model are within a couple of orders of magnitude of the TeV scale. Randall and Sundrum were right!