Physics 690/482-02, Spring 2005 Josh Erlich

Problem Set 10: Scalar branes and the Bianchi identity

Consider the action for a scalar field coupled to gravity in $d+1$ dimensions:

$$S = \int d^{d+1}x\,\sqrt{|g|}\,\left[(2M^{d-1})R - \frac{1}{2}(\partial_M\phi)^2 + V(\phi)\right]\ .$$

The stress tensor is,

$$T_{MN} = -\frac{1}{\sqrt{|g|}}\frac{\delta\mathcal{L}}{\delta g^{MN}} = \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{2}g_{MN}\left[-\frac{1}{2}(\partial_P\phi)^2 + V(\phi)\right]\ .$$

Assume a solution to Einstein's equations and the scalar field equation of motion of the form:

$$\begin{aligned} ds^2 &= e^{-A(y)}dx_4^2 + dy^2 \\ \phi &= \phi(y) \end{aligned}$$

1. Evaluate the components of T_{MN} given the ansatz for the metric and scalar field above.

2. In Problem Set 8 you worked out the Christoffel symbols for the warped metric. The covariant derivative of the stress tensor is,

$$D_M T^{NP} = \partial_M T^{NP} + \Gamma^N_{AM}T^{AP} + \Gamma^P_{AM}T^{NA}\ .$$

Evaluate the covariant divergence of the stress tensor, $D_M T^{MN}$.

3. In Problem Set 8 you also worked out the Einstein tensor, $R_{MN} - 1/2\,g_{MN}R$, in the warped geometry. What are Einstein's equations with the stress tensor from Problem 1? (*Hint*: You should get the result quoted in class.)

4. What is the scalar field equation of motion (*i.e.* the Euler-Lagrange equation for ϕ), assuming our ansatz for the metric and ϕ?

5. The Bianchi identity requires that the stress tensor be covariantly conserved, *i.e.* $D_M T^{MN} = 0$. Show that with our ansatz for the metric and scalar field, if we also assume that $\phi'(y) \neq 0$ then the Bianchi identity implies the scalar field equation of motion.