Physics 690/482-02 Spring, 2005 Josh Erlich
Notes: History of Extra Dimensions

In this course we will study physics in extra dimensions. The idea of extra dimensions goes back a long time, but the formalism to describe geometry of arbitrary dimensional manifolds is only a century and a half old, dating back to the 1850's with the work of **Bernhard Riemann**. Before Riemann there was little serious thought given to the possibility of extra spatial dimensions, although that possibility had sometimes been used as an explanation of magic and witchcraft.

A good place to start in discussing geometry is the work of the early Mediterranean mathematician-philosophers. **Pythagoras of Samos** (c. 569-475 b.c.) proved the famous theorem that the three sides of a right triangle satisfy the relation $c^2 = a^2 + b^2$ where a, b and c are the lengths of the sides of a right triangle, with c corresponding to the hypotenuse.

A century after Pythagoras, **Aristotle** pondered the concept of dimension, and concluded in *On Heaven* that, "the line has magnitude one way, the plane in two ways and the solid in three ways, and beyond these there is no other magnitude because the three are all." In a more modern language we would say that a line is a one-dimensional manifold, a plane is a two-dimensional manifold, and so on. Aristotle concluded from physical experience that there are no physical objects described by manifolds of dimension greater than three.

From there, the story of geometry has a rich history, but before jumping in time nearly two millenia we should recall **Euclid**'s (c. 325-275 b.c) role in formalizing what is now known as Euclidean geometry. But Euclid's geometry described what by modern accounts are rather uninteresting manifolds. Euclidean geometry is flat in a sense that will be explained.

In the 17th Century, **René Descartes** (1595-1650 a.d.) developed a method to graph points and lines on the plane by labeling points by a pair of numbers representing distances from some specified point along two orthogonal directions. This coordinate system is now commonly referred to as **Cartesian** in honor of Descartes. More generally, in d-dimensional Euclidean geometry, points labeled by a set of d numbers representing distance from a specified point in d mutually orthogonal directions, are also said to be specified in Cartesian coordinates.

Almost immediately following Riemann's pioneering work in the 1850's, in the late 1800's a number of mathematicians, such as **Charles Hinton** and **Edwin Abbott** became intrigued by the idea of extra dimensions and helped to popularize the concept. Abbott's book, *Flatland: A Romance of Many Dimensions* is a classic that deserves to be read by everyone interested in pondering the possibility of extra dimensions.

All of this is the backdrop for Einstein's theories of special and general relativity. In special relativity, as opposed to **Isaac Newton's** (1643-1727) view of the world, time and space are not absolute concepts. If one observer uses a set of rulers and clocks to describe events in space and time, then a second observer moving with respect to the first will find that using her otherwise identical rulers and clocks, the same events appear to take place at different positions and times. The transformation between the two observers' coordinate systems is called a **Lorentz transformation** after **Hendrik A. Lorentz** (1853-1928), and mixes up measurements of locations and time.

The concept of space-time was invented by **Hermann Minkowski** in 1909 when he realized that if he formally thought of time as an imaginary coordinate, so that events are points in a four-dimensional **space-time** with coordinates (x_1, x_2, x_3, ict) (where c is the speed of light), then Lorentz transformations can be thought of as rotations in space-time. Furthermore, Maxwell's theory of electromagnetism took a particularly simple form in the language of space-time. Today, all models of particle physics and gravity can be described in terms of fields living in space-time.

In 1915 Einstein completed his theory of **general relativity**, in which the **equivalence principle**, the principle that all bodies should fall the same way in a gravitational field, was explained by postulating that bodies move along trajectories which minimize the distance between points in space-time. So the motion of an object falling under gravity depends only on the geometry of space-time, and not on the composition of that object. But what determines the geometry of space-time? That depends on the energy and momentum carried by all the matter in the universe.

Now back to extra dimensions....

In 1914 **Gunnar Nordstrom** discovered that he could unite the physics of electromagnetism and gravity by postulating the existence of a fourth spatial dimension. However, recall that general relativity did not exist yet, so Nordstrom had not produced the correct theory of gravity, but only an approximation.

Theodor Kaluza discovered in 1921 that by making certain assumptions, the equations of general relativity with an extra spatial dimension contains Maxwell's equations for an electromagnetic field. However, there was no explanation given for Kaluza's *ad hoc* assumption that none of the fields in the universe should vary over the extra dimension. In 1926 **Oskar Klein** provided an explanation for Kaluza's assumption, namely that if the extra dimension forms a circle much smaller than the distance scales we commonly observe, then as long as a field begins in a state which is constant in the extra dimension it will stay that way. Quantum mechanically, the statement is that all energy eigenstates will have masses which are inversely proportional to the size of the circle, and there's no way to produce such states without a lot of energy. The theory of gravity on a compact space-time is called **Kaluza-Klein theory**.

Kaluza-Klein theory still had certain problems in its interpretation as a theory which united gravity and electromagnetism, most notably that it predicted a unit of charge which was many times smaller than the electron's charge. As a result, many physicists left the idea of extra dimensions for the realm of curiosity, until...

In the 1960's **string theory** was invented as a model of the strong interactions which bind nuclei together. The idea was that the zoo of particles that were being discovered in particle accelerators were to be thought of as vibrations of a relativistic string. The theory had several problems, though: it had a tachyon, which signalled an instability; it had a massless spin-two field in the spectrum that was not observed in accelerators; and worst of all the theory seemed to suffer from an anomaly which would have made the entire theory inconsistent.

Then in 1984 **Michael Green** and **John Schwartz** discovered that in fact there was no anomaly in the theory so the theory might be consistent, and together with other physicists it was discovered that the instability mentioned above could be removed by making the theory **supersymmetric**. All this made sense, but only in 10 dimensions (or a combination of 10 and 26 dimensions). That's okay, though, because Kaluza and Klein already taught us that if all of the dimensions are compact and small enough, then we would not have noticed them yet. But what of that massless spin-two field? Well, that is precisely the type of field that could describe gravity, and so string theory became a quantum theory of gravity. This sequence of events has become known as "the first string revolution."

A decade later, **Joe Polchinski** and others discovered that string theory contained in it objects which extended in various numbers of dimensions. These objects are called p-branes if they extend in p spatial dimensions. It was found that many such objects have the property that open strings can end on them, and are thereby stuck to a manifold with p spatial dimensions. Studying quantum mechanics of a string stuck to a brane it was found that the oscillations of the string contain a massless particle that could be interpreted as the electromagnetic field.

This suggests another way in which we might not see the presence of extra dimensions – for some reason or another, we might just be stuck to a **braneworld** that extends in three spatial dimensions. In 1996, **Petr Horava** and **Edward Witten** studied a (10+1)-dimensional theory of gravity in which one of the spatial dimensions exists only in an interval bounded by two 9-branes. This theory was related to one of the five consistent string theories that have been developed. Two years later, **Nima Arkani-Hamed**, **Savas Dimopoulos** and **Georgi Dvali** (ADD) pointed out that in a similar scenario, where we are stuck to a brane which extends in three spatial dimensional brane sitting in a higher-dimensional space-time, the extra dimensions might have been of much larger spatial extent than we have studied in particle physics experiments and we would still not have seen them in *any* experiment or observation that had been reported by that time. Since then, particle physics and gravity experiments have been able to place constraints on the sizes of these dimensions. Furthermore, ADD spearheaded a line of study in answer to the question, "what are extra dimensions good for?"

In 1999, **Lisa Randall** and **Raman Sundrum** pointed out that even if an extra dimension is infinitely long, in a **warped space-time geometry** a braneworld might still mimic ordinary (3+1)-dimensional gravity. The reason this was surprising is that it had generally been believed that the larger the size of the extra dimension, the weaker gravity would seem to the braneworld observer. So it was thought that an infinitely large extra dimension would imply that gravity be infinitely weak, *i.e.* decoupled from the braneworld. But in a warped extra dimensional scenario that need not be true. The model of Randall and Sundrum has been generalized in many ways over the past five years in order to apply the basic scenario to various problems in particle physics and cosmology. We will spend a lot of time in this course studying various scenarios along these lines.

References

[1] http://www-groups.dcs.st-and.ac.uk/ history/Mathematicians/
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[2] http://www.tech.port.ac.uk/staffweb/seahras/neat_physics/extra_dimensions/contents.htm
A brief history of extra dimensions from Sanjeev Seahra's thesis at U. Waterloo