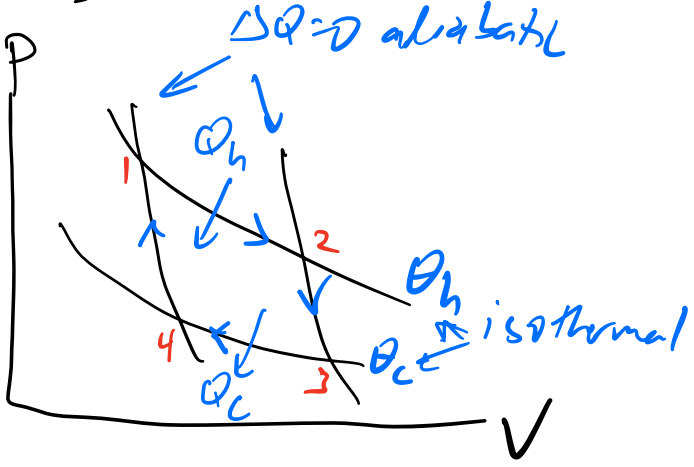
PHYS 630 S'24 Homework 1 Solutions

Kardar 1.3

a)



Carnot cycle

$$dQ = dE - dW = dE + P\,dV$$

$$E = E(\theta) \Rightarrow dQ = \frac{dE}{d\theta}\,d\theta + P\,dV.$$

Isothermal transitions: $d\theta = 0$

$$dQ = P\,dV \quad , \quad P = \frac{N k_B \theta}{V} \text{ for ideal gas}$$

$$\Rightarrow Q_h = \int_{V_1}^{V_2} N k_B \theta_h \frac{dV}{V} = N k_B \theta_h \ln \frac{V_2}{V_1}$$

$$Q_c = -\int_{V_3}^{V_4} N k_B \theta_c \frac{dV}{V} = N k_B \theta_c \ln \frac{V_3}{V_4}$$

↑ heat flow out of system

$$\Rightarrow \boxed{\frac{Q_h}{Q_c} = \frac{\theta_h}{\theta_c} \frac{\ln V_2/V_1}{\ln V_3/V_4}}$$

b) Adiabatic: $dQ = 0$

$$0 = \frac{dE}{d\theta} d\theta + \frac{N k_B \theta}{V} dV$$

$$\frac{dV}{V} = -\frac{1}{N k_B} E'(\theta) \frac{d\theta}{\theta}$$

Integrate: $$\ln \frac{V_3}{V_2} = -\frac{1}{N k_B} \int_{\theta_c}^{\theta_h} \frac{E'(\theta)}{\theta} d\theta$$

$$\ln \frac{V_4}{V_1} = -\frac{1}{N k_B} \int_{\theta_c}^{\theta_h} \frac{E'(\theta)}{\theta} d\theta$$

$$= \ln \frac{V_3}{V_2}$$

$$\Rightarrow \boxed{\frac{V_4}{V_1} = \frac{V_3}{V_2}} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

part (b)

c) From part (a), $$\frac{Q_h}{Q_c} = \frac{\theta_h}{\theta_c} \frac{\ln \frac{V_2}{V_1}}{\ln \frac{V_3}{V_4}} = \frac{\theta_h}{\theta_c}$$

Thermodynamic temperature scale defined by Carnot cycle: $\frac{Q_h}{Q_c} = \frac{T_h}{T_c} \Rightarrow \frac{\theta_h}{\theta_c} = \frac{T_h}{T_c} \Rightarrow \boxed{\theta \propto T}$