Physics 601, Fall 2021
Homework 2, due Monday, September 27.

1. *Problems from Goldstein, Poole & Safko*

Chapter 1: 1.9, 1.14

Chapter 2: 2.12, 2.20

2. *Noether's Theorem*

Consider a particle moving in a radial potential $V(|\mathbf{r}|)$, where $|\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$, with no other forces acting on the particle.

A rotation about the z-axis acts on the particle position by the transformation,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} \cos\epsilon & -\sin\epsilon & 0 \\ \sin\epsilon & \cos\epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

a) Show that the Lagrangian $L = 1/2\, m\, \dot{\mathbf{r}}^2 - V(|\mathbf{r}|)$ is invariant under the rotation.

b) Identify Δx, Δy, and Δz, where for small rotation angle ϵ the rotation acts by $x \rightarrow x + \epsilon\, \Delta x + \mathcal{O}(\epsilon)^2$, and analogously for y and z.

c) Use Noether's theorem to identify the conserved quantity associated with invariance under rotations about the z-axis.