# Phys 201 F'09 Problem Set 8 Solutions

5.10 a) The energy of an electron in the $n^{th}$ orbit of Hydrogen has energy $E_n = -\frac{E_R}{n^2}$ w/ $E_R = \frac{m_e (ke^2)^2}{2\hbar^2}$.

Since the total energy is $E = \underset{\text{kinetic}}{K} + \underset{\text{potential}}{U}$ and $K = -\frac{1}{2}U$,

$$E = -K = -\frac{1}{2} m_e V^2$$

$$-\frac{1}{2} m_e V_n^2 = -\frac{m_e (ke^2)^2}{2\hbar^2 n^2}$$

$$\boxed{V_n = \frac{ke^2}{\hbar n}}$$

b) Setting $n=1$, $v_1 = \frac{ke^2}{\hbar} = \frac{ke^2}{\hbar c} \cdot c = \frac{1.44 \text{ eV}\cdot\text{nm}}{197 \text{ eV}\cdot\text{nm}} c$

$$= \boxed{7.31 \times 10^{-3} \; c}$$

c) $\alpha = \frac{ke^2}{\hbar c} = 7.31 \times 10^{-3}$

$\frac{1}{137} = 7.30 \times 10^{-3}$, so $\boxed{\alpha \approx \frac{1}{137}}$

5.13 a) $r_1 = \frac{\hbar^2}{ke^2 m_\mu} = \frac{\hbar^2}{ke^2 m_e} \frac{m_e}{m_\mu} = a_B \cdot \frac{1}{207}$

$$= (0.0529 \text{ nm}) \cdot \frac{1}{207} = \boxed{2.56 \times 10^{-4} \text{ nm}}$$

$$E_1 = -\frac{m_\mu (ke^2)^2}{2\hbar^2} = -\frac{m_\mu}{m_e} \cdot \frac{m_e (ke^2)^2}{2\hbar^2} = -207 \, E_R$$

$$= -207 (13.6 \text{ eV}) = \boxed{-2815 \text{ eV}}$$

b) $E_\gamma = -E_1 \left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$, Lyman-$\alpha$ $\Rightarrow n'=1, n=2$

$E_\gamma = 2815 \text{ eV} \left(1 - \frac{1}{4}\right) = \frac{3}{4} \cdot 2815 \text{ eV} = 2110 \text{ eV}$

$\lambda = \frac{hc}{E_\gamma} = \frac{1240 \text{ eV}\cdot\text{nm}}{2110 \text{ eV}} = \boxed{0.59 \text{ nm}}$,

which is in the $\boxed{\text{X-ray}}$ or deep UV

6.1 $\lambda = \frac{h}{p} = \frac{6.6\times10^{-34}\ \mathrm{J\cdot s}}{(10^{-6}\ \mathrm{kg})(10^{-2}\ \mathrm{m/s})} = \boxed{6.6\times10^{-26}\ \mathrm{m}}$

not detectable

6.15 $p = \sqrt{2mK} = \sqrt{2(0.511\times10^{6}\ \mathrm{eV/c^2})\cdot 3\,\mathrm{eV}} = 1750\ \mathrm{eV/c}$

$\lambda = \frac{h}{p} = \frac{hc}{1750\,\mathrm{eV}} = \frac{1240\ \mathrm{eV\cdot nm}}{1750\ \mathrm{eV}} = 0.71\ \mathrm{nm}$

Constructive interference: $d\sin\theta = n\lambda$

First maximum: $n=1 \longrightarrow d = \lambda/\sin\theta$

$d = \frac{0.71\ \mathrm{nm}}{\sin 15^\circ} = \boxed{2.74\ \mathrm{nm}}$

6.21 $\lambda = 550\ \mathrm{nm} \longrightarrow k = \frac{2\pi}{\lambda} = \boxed{0.01\ \mathrm{rad/nm}}$

$\omega = 2\pi f = 2\pi\frac{c}{\lambda} = 2\pi\frac{3\cdot10^{8}\ \mathrm{m/s}}{550\times10^{-9}\ \mathrm{m}} = \boxed{3.4\times10^{15}\ \mathrm{rad/s}}$

6.33 $P(x) = \begin{cases} \frac{1}{2a} & -a < x < a \\ 0 & |x| \geq a \end{cases}$

$\Delta x = \sqrt{\int_{-\infty}^{\infty} x^2 P(x)\,dx} = \sqrt{\int_{-a}^{a} x^2 \cdot \frac{1}{2a}\,dx}$

$= \sqrt{\frac{1}{2a}\cdot\frac{x^3}{3}\Big|_{-a}^{a}} = \sqrt{\frac{1}{2a}\cdot\left(\frac{a^3}{3} - \frac{(-a^3)}{3}\right)}$

$= \sqrt{\frac{a^2}{3}} = \boxed{\frac{a}{\sqrt{3}}}$