Phys 201 F'09 Problem Set 7 Solutions

4.4 a) $I(T) = \int_0^\infty d\lambda \; I(\lambda, T)$

$$= \int_0^\infty d\lambda \; \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

Change variables:

$$\lambda = \frac{hc}{x k_B T}, \quad d\lambda = -\frac{hc\,dx}{x^2 k_B T}, \quad \begin{matrix} \lambda = 0 \to x = \infty \\ \lambda = \infty \to x = 0 \end{matrix}$$

$$I(T) = \int_0^\infty dx \; \frac{hc}{x^2 k_B T} \cdot 2\pi h c^2 \left(\frac{x k_B T}{hc}\right)^5 \left(e^x - 1\right)^{-1}$$

$$= \underbrace{2\pi h^{-3} c^{-2} k_B^4 \int_0^\infty dx \; x^3 \left(e^x - 1\right)^{-1}}_{\sigma} \; T^4$$

b) $\sigma = 2\pi h^{-3} c^{-2} k_B^4 \left(\pi^4/15\right)$

$$= \frac{2\pi^5 k_B^4}{15 h^3 c^2}$$

c) $$\sigma = \frac{2\pi^5 \left(1.38\times10^{-23}\ \mathrm{J/K}\right)^4}{15\left(6.63\times10^{-34}\ \mathrm{Js}\right)^3 \left(3\times10^8\ \mathrm{m/s}\right)^2} = \boxed{5.64\times10^{-8}\ \mathrm{J/s}\ \mathrm{m}^{-2} K^{-4}}$$

$$I(1000°K) = \left(5.64\times10^{-8}\ \mathrm{J/s}\ \mathrm{m}^{-2} K^{-4}\right)\left(10^3 K\right)^4$$

$$= 5.64\times10^4 \ \frac{\mathrm{J}}{\mathrm{s\,m^2}}$$

$$r = 1\,\mathrm{cm} \to A = 4\pi r^2 = 4\pi\,(0.01\,\mathrm{m})^2 = 1.3\times10^{-3}\ \mathrm{m}^2$$

Total radiated power $= I(1000°K)\ A$

$$= \left(5.64\times10^4 \ \frac{\mathrm{J}}{\mathrm{s\,m^2}}\right)\left(1.3\times10^{-3}\ \mathrm{m}^2\right)$$

$$= \boxed{73\ \mathrm{J/s}}$$

4.8 a) $E = \frac{hc}{\lambda} = \frac{1240\ eV \cdot 10^{-9} m}{0.01\ m} = \boxed{1.24 \times 10^{-4}\ eV}$

b) $E = K_B T = (8.62 \times 10^{-5}\ eV/K)(3\ K) = 2.59 \times 10^{-4}\ eV$

$\lambda = \frac{hc}{E} = \frac{1240\ eV \cdot 10^{-9} m}{2.59 \times 10^{-4}\ eV} = \boxed{4.79 \times 10^{-3}\ m}$

4.12 Max Kinetic energy of ejected electron $K_{max} = hf - \phi = \frac{hc}{\lambda} - \phi$

Max Wavelength that can eject electron: $K_{max} = 0$

$$\frac{hc}{\lambda_{max}} = \phi \implies \lambda_{max} = \frac{hc}{\phi}$$

a) $\lambda_{max} = \frac{1240\ eV \cdot nm}{1.9\ eV} = \boxed{650\ nm}$

b) $K_{max} = \frac{1240\ eV \cdot nm}{500\ nm} - 1.9\ eV = \boxed{0.6\ eV}$

4.14 a) Energy of each photon $E_\gamma = \frac{hc}{\lambda} = \frac{1240\ eV \cdot nm}{550\ nm} = 2.25\ eV$

$\#\ photons/sec = \frac{(3\ J/s)}{2.25\ eV} \cdot \frac{1\ eV}{1.6 \times 10^{-19}\ J} = \boxed{8.3 \times 10^{18}\ photons/sec}$

b) At a distance of 10 ft from the bulb, the photons are uniformly distributed on a sphere of area

$A_{sphere} = 4\pi (10\ ft)^2 = 4\pi (3048\ mm)^2 = 1.17 \times 10^8\ mm^2$

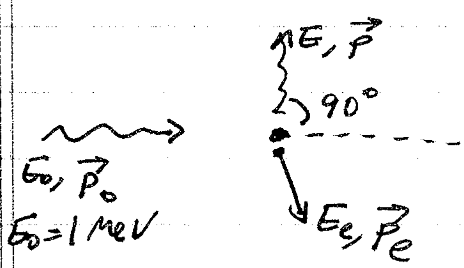
Fraction of photons entering eye $= \frac{A_{pupil}}{A_{sphere}} = \frac{\pi (0.5\ mm)^2}{1.17 \times 10^8\ mm^2}$

$= 6.7 \times 10^{-9}$

$\#\ photons\ entering\ eye/sec = (8.3 \times 10^{18}\ photons/sec)(6.7 \times 10^{-9}) = \boxed{5.6 \times 10^{10}\ /sec}$

c) This is more than $\boxed{\text{8 orders of magnitude}}$ above the minimum detectable intensity.

4.27



$$\frac{1}{p} - \frac{1}{p_0} = \frac{1}{mc}(1 - \cos 90°) = \frac{1}{mc}$$

$$p_0 = E_0/c = 1 \text{ MeV}/c \;,\; mc = 0.511 \text{ MeV}/c$$

$$\frac{1}{p} = \frac{1}{1 \text{ MeV}/c} + \frac{1}{0.511 \text{ MeV}/c} = 2.96 \cdot \frac{c}{\text{MeV}} \;,\; p = 0.34 \frac{\text{MeV}}{c}$$

$$E = pc = 0.34 \text{ MeV}$$

$$E_{initial} = E_0 + mc^2$$
$$E_{final} = E + mc^2 + K$$

(K ← kinetic energy of electron)

$$K = E_0 - E = 1 \text{ MeV} - 0.34 \text{ MeV} = \boxed{0.66 \text{ MeV}}$$

5.3 $E_\gamma = hcR\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$

Max energy: $n' = 1, n \to \infty$

$$E_{max} = hcR = (1240 \text{ eV}\cdot\text{nm})(0.0110 \text{ nm}^{-1}) = \boxed{13.6 \text{ eV}}$$

Min energy: $n', n \to \infty$, $\boxed{E_{min} = 0}$

5.4 $ke^2 = (8.99 \times 10^9 \text{ Nm}^2/C^2)(1.6 \times 10^{-19} C)^2$

$$= 2.30 \times 10^{-28} \text{ N}\cdot\text{m}^2 = 2.30 \times 10^{-28} \text{ J}\cdot\text{m}$$

$$= (2.30 \times 10^{-28} \text{ J}\cdot\text{m}) \cdot \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \cdot \frac{10^9 \text{ nm}}{1 \text{ m}}$$

$$= \boxed{1.44 \text{ eV nm}}$$

$$hc = (6.63 \times 10^{-34} \text{ Js})(3 \times 10^8 \text{ m/s})$$

$$= (1.99 \times 10^{-25} \text{ J m}) \cdot \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \cdot \frac{10^9 \text{ nm}}{1 \text{ m}}$$

$$= \boxed{1240 \text{ eV}\cdot\text{nm}}$$