Phys 201 F'09 Problem Set 4 Solutions

2.25 a) $E_{initial} = 2 m_{H_2} c^2 + m_{O_2} c^2$

$E_{final} = 2\, m_{H_2O} c^2 + 5eV$

$E_{initial} = E_{final} \longrightarrow \underbrace{2 m_{H_2} c^2 + m_{O_2} c^2 - 2 m_{H_2O} c^2}_{\Delta m\, c^2} = 5eV$

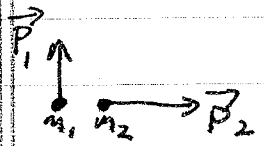
$$\boxed{\Delta m = 5eV/c^2}$$

b) $m_{H_2O} = \dfrac{3\times10^{-26}\ \text{Kg}}{1.783\times10^{-36}\ \text{Kg}/eV/c^2} = 1.7\times10^{10}\ eV/c^2$

$\dfrac{\Delta m}{\text{Total Mass}} = \dfrac{\Delta m}{2 m_{H_2O}} = \dfrac{5eV/c^2}{2\times1.7\times10^{10}\ eV/c^2} = \boxed{1.5\times10^{-10}}$

c) Change in mass $= 10g \times \dfrac{\Delta m}{\text{Total Mass}} = \boxed{1.5\times10^{-9}\ g}$

2.31



$m_1 = 0.5\ GeV/c^2,\quad \vec{P}_1 = 2\ GeV/c\ \hat{y}$

$m_2 = 1\ GeV/c^2,\quad \vec{P}_2 = 1.5\ GeV/c\ \hat{x}$

$\vec{P}_{Tot} = \vec{P}_1 + \vec{P}_2 = \frac{3}{2}\ GeV/c\ \hat{x} + 2\ GeV/c\ \hat{y}$

$P_{Tot} \equiv |\vec{P}_{Tot}| = \sqrt{(\frac{3}{2})^2 + 2^2}\ GeV/c = \frac{5}{2}\ GeV/c$

$E_{Tot} = \sqrt{P_1^2c^2 + m_1^2c^4} + \sqrt{P_2^2c^2 + m_2^2c^4}$

$= \sqrt{2^2 + (\frac{1}{2})^2}\ GeV + \sqrt{(\frac{3}{2})^2 + 1}\ GeV$

$= \frac{1}{2}(\sqrt{17} + \sqrt{13})\ GeV = 3.86\ GeV$

Mass of decaying particle $M = \sqrt{E_{Tot}^2 - P_{Tot}^2c^2} \cdot \dfrac{1}{c^2}$

$= \sqrt{3.86^2 - 2.5^2}\ GeV/c^2$

$= \boxed{2.95\ GeV/c^2}$

Speed of decaying particle $u = \dfrac{P_{Tot}c^2}{E_{Tot}} = \dfrac{2.5\ GeV/c \cdot c^2}{3.86\ GeV}$

$= \boxed{0.65\,c}$

2.35 a) We will need $\frac{d\gamma}{dt}$!

$$\frac{d\gamma}{dt} = \frac{d}{dt}\left[\left(1-\frac{\vec{u}^2}{c^2}\right)^{-1/2}\right] = -\frac{1}{2}\left(1-\frac{\vec{u}^2}{c^2}\right)^{-3/2}\cdot\frac{(-2)}{c^2}\vec{u}\cdot\frac{d\vec{u}}{dt}$$

$$= \gamma^3 \frac{\vec{u}}{c^2}\cdot\frac{d\vec{u}}{dt}$$

$$\vec{u}\perp\vec{F} \Longrightarrow \vec{u}\cdot\vec{F}=0$$

$$\vec{u}\cdot\vec{F} = \vec{u}\cdot\frac{d\vec{p}}{dt} = \vec{u}\cdot\frac{d}{dt}(m\vec{u}\gamma)$$

$$= m\gamma\vec{u}\cdot\frac{d\vec{u}}{dt} + m\vec{u}^2\frac{d\gamma}{dt}$$

$$= m\gamma\vec{u}\cdot\frac{d\vec{u}}{dt} + m\vec{u}^2\gamma^3\frac{\vec{u}}{c^2}\cdot\frac{d\vec{u}}{dt}$$

$$= m\gamma^3\vec{u}\cdot\frac{d\vec{u}}{dt}\left(\gamma^{-2}+\frac{\vec{u}^2}{c^2}\right)$$

$$= m\gamma^3\vec{u}\cdot\frac{d\vec{u}}{dt}\left(1-\frac{\vec{u}^2}{c^2}+\frac{\vec{u}^2}{c^2}\right)$$

$$= m\gamma^3\vec{u}\cdot\frac{d\vec{u}}{dt}$$

$$= mc^2\frac{d\gamma}{dt} = 0 \longrightarrow \frac{d\gamma}{dt} = 0$$

Then $\vec{F} = m\gamma\frac{d\vec{u}}{dt} + m\vec{u}\cancelto{0}{\frac{d\gamma}{dt}} = m\gamma\frac{d\vec{u}}{dt}$

$$\boxed{\vec{F} = m\gamma\vec{a}}$$

b) $\vec{u}\parallel\vec{F}$ → The problem is one-dimensional.

$$F = m\frac{d}{dt}(u\gamma) = m\gamma\frac{du}{dt} + mu\frac{d\gamma}{dt}$$

$$= m\gamma\frac{du}{dt} + mu\gamma^3\frac{u}{c^2}\frac{du}{dt}$$

$$= m\gamma^3\frac{du}{dt}\left(\gamma^{-2}+u^2/c^2\right)$$

$$= m\gamma^3 a\left(1-u^2/c^2+u^2/c^2\right) = m\gamma^3 a$$

$$\Rightarrow \boxed{\vec{F} = m\gamma^3\vec{a}}$$

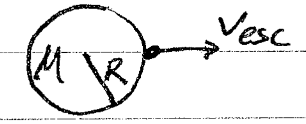
2.38 $\quad (\pi^0) \longrightarrow \vec{u}_\pi$

$\overleftarrow{\sim\sim}\ \vec{p}_1 \qquad \overrightarrow{\sim\sim}\ \vec{p}_2 \qquad |\vec{p}_2|c = 3|\vec{p}_1|c \longrightarrow |\vec{p}_2| = 3|\vec{p}_1|$

Conservation of Momentum: $|\vec{p}_\pi| = |\vec{p}_1 + \vec{p}_2| = -|\vec{p}_1| + 3|\vec{p}_1|$
$= 2|\vec{p}_1|$

Conservation of Energy: $E_\pi = |\vec{p}_1|c + 3|\vec{p}_1|c = 4|\vec{p}_1|c$

$$u_\pi = \frac{|\vec{p}_\pi|c^2}{E_\pi} = \frac{2|\vec{p}_1|c^2}{4|\vec{p}_1|c} = \boxed{\tfrac{1}{2}c}$$

2.46 a) 

At infinity, $V \to 0$, $\frac{1}{2}mV^2 \to 0$, $U = -\frac{GMm}{r} \to 0$ $\Big\}\ E = 0$

$$E = \frac{1}{2}mV_{esc}^2 - \frac{GMm}{R} = 0$$

$$R = \frac{2GM}{V_{esc}^2}$$

$$V_{esc} = c \longrightarrow \boxed{R_s = \frac{2GM}{c^2}}$$

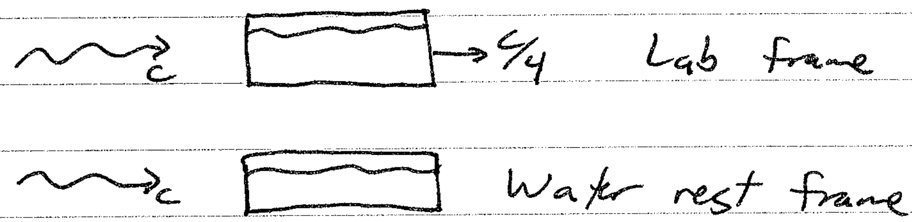
b) $$R_s = \frac{2(6.67\times10^{-11}\ Nm^2/Kg^2)(20\times10^{30}\ Kg)}{(3\times10^8\ m/s)^2}$$

$$\approx \boxed{30\ Km}$$

c) Density $$\rho = \frac{M}{\frac{4}{3}\pi R_s^3} = \frac{20\times10^{30}\ Kg}{\frac{4}{3}\pi(3\times10^4\ m)^3}$$

$$= \boxed{1.8\times10^{17}\ Kg/m^3}$$

A1.



In the water's rest frame, the index of refraction $n = 4/3$ is the ratio of c to the speed of light in water u'_{water}, i.e. $u'_{water} = \frac{3c}{4}$.

In the lab frame, $u_{water} = \dfrac{u'_{water} + c/4}{1 + \frac{u'_{water}\, c/4}{c^2}}$

$$= \frac{3c/4 + c/4}{1 + 3c/4 \cdot c/4 \cdot \frac{1}{c^2}} = \boxed{\frac{16}{19} c}$$

A2. $E = 3mc^2 = \sqrt{p^2c^2 + m^2c^4}$

$$p = \frac{1}{c}\sqrt{E^2 - m^2c^4} = \frac{1}{c}\sqrt{(3mc^2)^2 - m^2c^4}$$

$$= \boxed{2\sqrt{2}\, mc}$$