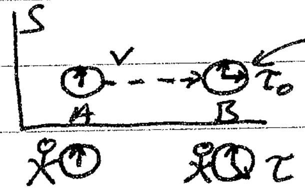
Phys 201 F'09 Problem Set 2 Solutions

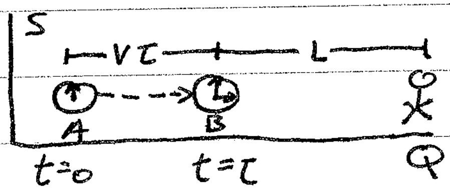
1.28 a)



physical clock carried from A to B

τ = time at which the physical clock is at B as measured in frame S.

$= \gamma \tau_0$



L = distance from B to Q in frame S

Time at which light from A reaches Q $= \frac{L+v\tau}{c}$

Time at which light from B reaches Q $= \frac{L}{c}+\tau$

$$\tau_{see} = \left(\frac{L}{c}+\tau\right)-\left(\frac{L+v\tau}{c}\right)$$

$$= \tau - \frac{v}{c}\tau$$

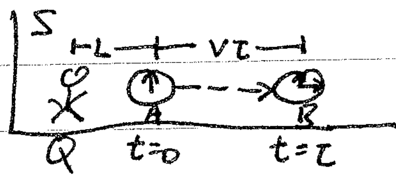
$$= \tau(1-\beta)$$

$$= \tau_0\sqrt{\frac{1}{1-\beta^2}}\,(1-\beta)$$

$$= \tau_0\sqrt{\frac{(1-\beta)^2}{(1-\beta)(1+\beta)}}$$

$$\boxed{\tau_{see} = \tau_0\sqrt{\frac{1-\beta}{1+\beta}}}$$

b)



Light from A reaches Q at time $\frac{L}{c}$

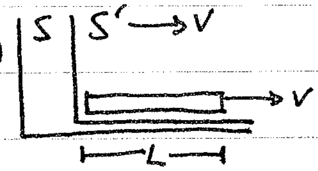
Light from B reaches Q at time $\frac{L+v\tau}{c}+\tau$

$$\tau_{see} = \left(\frac{L+v\tau}{c}+\tau\right)-\left(\frac{L}{c}\right)$$

$$= \tau(1+\beta)$$

$$= \tau_0\sqrt{\frac{(1+\beta)^2}{(1-\beta)(1+\beta)}}$$

$$\boxed{\tau_{see} = \tau_0\sqrt{\frac{1+\beta}{1-\beta}}}$$

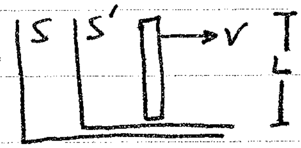
1.33 a) 

$$v = \frac{4}{5}c$$

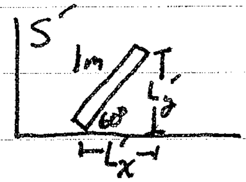
$$\gamma = \frac{1}{\sqrt{1-\frac{16}{25}}} = \frac{1}{\sqrt{\frac{9}{25}}} = \frac{5}{3}$$

$$L' = 1\,m$$

$$L = L'/\gamma = \boxed{\frac{3}{5}\,m} \quad \text{or} \quad \boxed{0.6\,m}$$

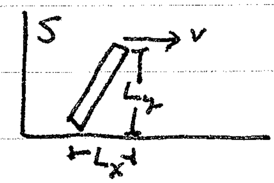
b) 

$$L = L' = \boxed{1\,m}$$

c) 

$$L_x' = (1\,m)\cos 60^\circ = \frac{1}{2}\,m$$

$$L_y' = (1\,m)\sin 60^\circ = \frac{\sqrt{3}}{2}\,m$$

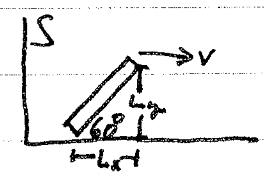


$$L_x = L_x'/\gamma = \frac{1}{2}\cdot\frac{3}{5}\,m = \frac{3}{10}\,m$$

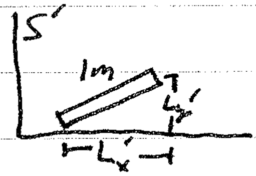
$$L_y = L_y' = \frac{\sqrt{3}}{2}\,m$$

$$L = \sqrt{L_x^2 + L_y^2} = \sqrt{\frac{9}{100}m^2 + \frac{3}{4}m^2} = \boxed{\frac{\sqrt{84}}{10}\,m}$$

$$\text{or } \boxed{0.92\,m}$$

d) 

$$L_y = L_x \tan 60^\circ = L_x\sqrt{3}$$



$$L_y' = L_y$$

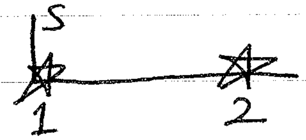
$$L_x' = L_x\gamma = \frac{5}{3}L_x$$

$$L' = \sqrt{L_x'^2 + L_y'^2} = \sqrt{\frac{25}{9}L_x^2 + L_y^2} = \sqrt{\frac{25}{9}L_x^2 + 3L_x^2} = 1\,m$$

$$\frac{\sqrt{52}}{3}L_x = 1\,m$$

$$L_x = \frac{3}{\sqrt{52}}\,m, \quad L_y = L_x\sqrt{3} = \frac{3\sqrt{3}}{\sqrt{52}}\,m$$

$$L = \sqrt{L_x^2 + L_y^2} = \sqrt{\frac{9}{52}m^2 + \frac{27}{52}m^2} = \boxed{\frac{6}{\sqrt{52}}\,m} \quad \text{or} \quad \boxed{0.83\,m}$$

1.41 

Event 1: $t=0,\ x_1=y_1=z_1=0$

Event 2: $t=0,\ x_2 = 4\ c\cdot yrs,\ y_2=z_2=0$

a) $S' \rightarrow v = \frac{3}{5}c$

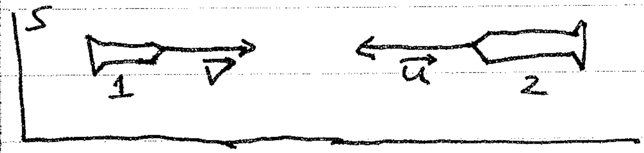
$\gamma = \frac{1}{\sqrt{1-(\frac{3}{5})^2}} = \frac{5}{4}$

$$\left.\begin{aligned} x_1' &= \gamma(x_1 - vt) = 0 \\ y_1' &= y_1 = 0 \\ z_1' &= z_1 = 0 \\ t_1' &= \gamma\left(t - \frac{v x_1}{c^2}\right) = 0 \end{aligned}\right\} \text{Event 1 in } S' \text{ frame}$$

$$\text{Event 2 in } S' \text{ frame} \left\{\begin{aligned} x_2' &= \gamma(x_2 - vt) = \frac{5}{4}\cdot 4\ c\cdot yrs = 5\ c\cdot yrs \\ y_2' &= y_2 = 0 \\ z_2' &= z_2 = 0 \\ t_2' &= \gamma\left(t - \frac{v x_2}{c^2}\right) = \frac{5}{4}\left(-\frac{(\frac{3}{5}c)\cdot 4\ c\cdot yrs}{c^2}\right) = -3\ yrs \end{aligned}\right.$$

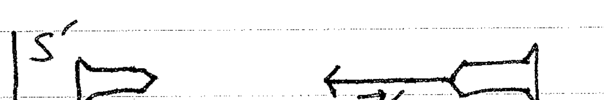
b) In the S' frame the two events are separated by $5\ c\cdot yrs$ $\quad (= x_2' - x_1')$

c) No. Event 2 occurs 3 yrs before event 1 in the S' frame.

1.46 

$\vec{V} = 0.9c\ \hat{x}$

$\vec{u} = -0.9c\ \hat{x}$



S' = rest frame of rocket 1.

$$u_x' = \frac{u_x - V}{1 - \frac{u_x V}{c^2}}$$

$$u_x' = \frac{-0.9c - 0.9c}{1 - \frac{(-0.9c)(0.9c)}{c^2}} = -\frac{1.8c}{1.81} = \boxed{-0.994\ c}$$