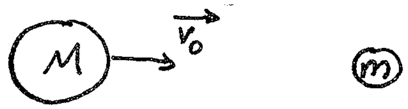
Phys 201 Fall '09 Problem Set 1 - Solutions

1.6



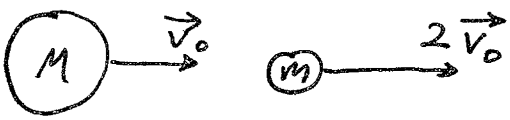
Lab frame before collision

To transform from lab frame to M's rest frame, add velocity $-\vec{v}_0$ to all velocities above:

M rest frame before collision

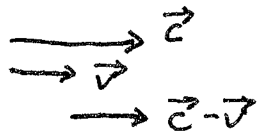
M rest frame after collision

To transform back to the lab frame, add velocity $\vec{v}_0$ to all velocities in M rest frame:



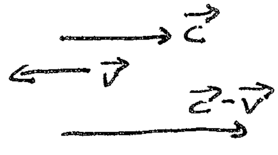
After the collision, m moves with speed $\underline{\underline{2v_0}}$ in the lab frame.

1.9 a)



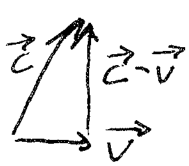
$$|\vec{c}-\vec{v}| = c - v = 2.9979\times10^8 \text{ m/s} - 0.0003\times10^8 \text{ m/s} = \boxed{2.9976\times10^8 \text{ m/s}}$$

b)



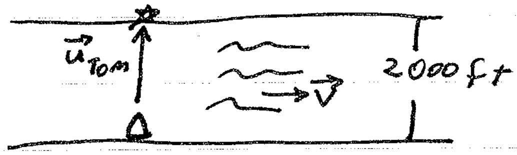
$$|\vec{c}-\vec{v}| = c+v = \boxed{2.9982\times10^8 \text{ m/s}}$$

c)



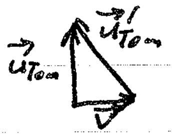
$$|\vec{c}-\vec{v}| = \sqrt{c^2-v^2} = c\sqrt{1-v^2/c^2}$$
$$\approx c\left(1-\tfrac{1}{2}v^2/c^2\right)$$
$$= \left(2.9979\times10^8 \text{ m/s}\right)\left(1-\tfrac{1}{2}\cdot10^{-8}\right)$$
$$\approx \boxed{2.9979\times10^8 \text{ m/s}}$$

1.15



$\vec{v}$ = velocity of water

$v = |\vec{v}| = 3$ ft/s



$\vec{u}'_{Tom} = \vec{u}_{Tom} - \vec{v}$ = Tom's velocity with respect to water's rest frame.

$|\vec{u}'_{Tom}| = 5$ ft/s

$$\vec{u}_{Tom} = \vec{u}'_{Tom} + \vec{v}$$

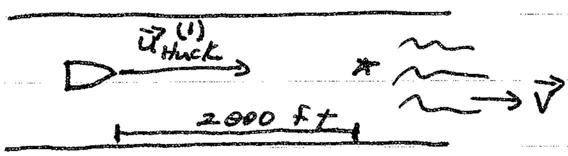
$$|\vec{u}_{Tom}| = \sqrt{(u'_{Tom})^2 - v^2}$$

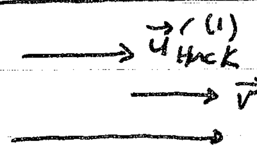
$$= \sqrt{(5\text{ ft/s})^2 - (3\text{ ft/s})^2} = 4\text{ ft/s}$$

Time for Tom to cross river $= \dfrac{2000\text{ ft}}{4\text{ ft/s}} = 500$ s

Time for Tom to return = 500 s as above.

Total time for Tom's trip = $\boxed{1000\text{ s}}$

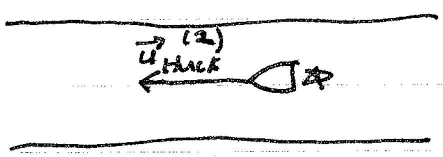




$$\vec{u}^{(1)}_{Huck} = \vec{u}'^{(1)}_{Huck} + \vec{v}$$

$$|\vec{u}^{(1)}_{Huck}| = |\vec{u}'^{(1)}_{Huck}| + v = 5\text{ ft/s} + 3\text{ ft/s} = 8\text{ ft/s}$$

Time for Huck to reach target $= \dfrac{2000\text{ ft}}{8\text{ ft/s}} = 250$ s



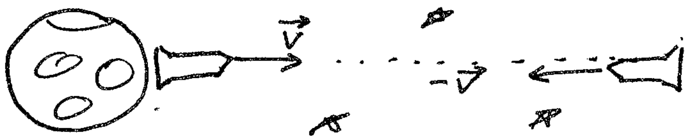
$$|\vec{u}^{(2)}_{Huck}| = 5\text{ ft/s} - 3\text{ ft/s} = 2\text{ ft/s}$$

(as above)

Time for Huck to return $= \dfrac{2000\text{ ft}}{2\text{ ft/s}} = 1000$ s

Total time for Huck's trip = $\boxed{1250\text{ s}}$

$\Rightarrow$ $\boxed{\text{Tom wins by 250 s!}}$

1.23 

Consider the clock in Mr. Spock's rocket.
The outgoing trip takes time $\frac{\Delta t}{2} = \frac{1}{2}$ week.

According to clocks on Earth, the trip takes time
$$\frac{\Delta t'}{2} = \gamma \frac{\Delta t}{2} = \frac{3}{2} \text{ weeks.}$$

The return trip is similar. Hence, $\gamma = 3$.
$$(1 - v^2/c^2)^{-1/2} = 3$$
$$1 - v^2/c^2 = \frac{1}{9}$$
$$v^2/c^2 = \frac{8}{9}$$
$$\frac{v}{c} = \frac{2\sqrt{2}}{3} = 0.94 \Rightarrow \boxed{v = 0.94c}$$

1.24 a) $\gamma = (1-\beta^2)^{-1/2} \approx 1 + \frac{1}{2}\beta^2$ using $(1-x)^p \approx 1 - px$.

b) $1/\gamma = (1-\beta^2)^{1/2} \approx 1 - \frac{1}{2}\beta^2$ similarly.

c) If $\beta = 1 - \varepsilon$ with $\varepsilon \ll 1$,
$$\gamma = \left(1 - (1-\varepsilon)^2\right)^{-1/2}$$
$$\approx \left(1 - (1 - 2\varepsilon)\right)^{-1/2} \quad \text{using } (1-\varepsilon)^2 \approx 1 - 2\varepsilon$$
$$= (2\varepsilon)^{-1/2}$$

1.26 a) $\gamma = (1-(0.8)^2)^{-1/2} = (0.36)^{-1/2} = \boxed{1.67}$

b) $t_{1/2}$ (rest frame) $= 1.8\times10^{-8}$ s

$t_{1/2}$ (lab frame) $= \gamma\, t_{1/2}$ (rest frame) $= (1.67)\times(1.8\times10^{-8}\text{ s})$

$= \boxed{3.0\times10^{-8}\text{ s}}$

c) $N_0 = 32{,}000$

t = Time to travel 36 m $= \dfrac{36\text{ m}}{(0.8)(3.0\times10^{8}\text{ m/s})} = 1.5\times10^{-7}$ s

Number of half-lives in lab frame $= \dfrac{t}{t_{1/2}(\text{lab})} = \dfrac{1.5\times10^{-7}\text{ s}}{3.0\times10^{-8}\text{ s}} = 5$

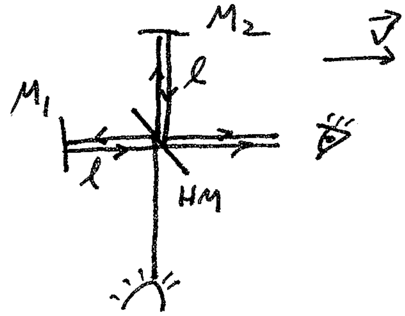
N = Number of pions surviving $= \dfrac{N_0}{2^5} = \dfrac{32{,}000}{32} = \boxed{1{,}000}$

d) Without time dilation, $t_{1/2}$ (lab frame) $= t_{1/2}$ (rest frame)

Number of half lives would be $\dfrac{1.5\times10^{-7}\text{ s}}{1.8\times10^{-8}\text{ s}} = 8.33$

$N = \dfrac{N_0}{2^{8.33}} = \dfrac{32{,}000}{322} \approx \boxed{99}$

Additional Problem



$l = 11\,m$

$v = 3\times10^4\,m/s$

$c = 3\times10^8\,m/s$ speed of light in ether frame.

In class we calculated the time difference between the length of the trip from HM to M_1 and back, and the length of the trip from HM to M_2 and back. We found for $\frac{v}{c} \ll 1$, $\boxed{\Delta t \approx \frac{l}{c}\cdot\frac{v^2}{c^2}}$.

This assumed (incorrectly) that the speed of light is c only in the ether frame, and velocities are added classically.

$$\Delta t \approx \frac{11\,m}{3\times10^8\,m/s}\cdot\left(\frac{3\times10^4\,m/s}{3\times10^8\,m/s}\right)^2 = 3.67\times10^{-16}\,s.$$

$$T = \text{Period of light} = \lambda/c = \frac{590\times10^{-9}\,m}{3\times10^8\,m/s} = 1.97\times10^{-15}\,s$$

$$\boxed{\frac{\Delta t}{T} = 0.19}$$

Note! By rotating the experiment Michelson and Morley would have observed a change in this phase shift by twice the amount calculated above. It was that change which was searched for but not found.