Phys 201 - Fall '09 Exam 2 Solutions.

1. Suppose $\Psi(x,t) = A\exp[i(kx-\omega t)]$

$$\frac{\partial \Psi}{\partial t} = -i\omega\Psi,$$

$$\frac{\partial \Psi}{\partial x} = ik\Psi, \qquad \frac{\partial^2 \Psi}{\partial x^2} = -k^2\Psi$$

$$E = \hbar\omega \rightarrow i\hbar\frac{\partial \Psi}{\partial t} = E\Psi$$

$$p = \hbar k \rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = \frac{p^2}{2m}\Psi$$

$$-i\alpha\hbar\frac{\partial \Psi}{\partial x} = \alpha p\Psi$$

Hence, the appropriate partial differential equation is,

$$\boxed{i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} - i\alpha\hbar\frac{\partial \Psi}{\partial x}}$$

2. Write $\Psi(x,t) = \varphi(t)\psi(x)$ for stationary state

Then $i\hbar\varphi'(t)\psi(x) = -\frac{\hbar^2}{2m}\varphi(t)\psi''(x) - i\alpha\hbar\varphi(t)\psi'(x)$

from problem 1.

Dividing by $\Psi(x,t) = \varphi(t)\psi(x)$,

$$i\hbar\frac{\varphi'(t)}{\varphi(t)} = -\frac{\hbar^2}{2m}\frac{\psi''(x)}{\psi(x)} - i\alpha\hbar\frac{\psi'(x)}{\psi(x)}$$

The left-hand side is a function of t only;

The right-hand side is a function of x only.

Hence, they each must equal the same constant E.

$$\Longrightarrow \boxed{-\frac{\hbar^2}{2m}\psi''(x) - i\alpha\hbar\psi'(x) = E\psi(x)}$$

3. a) Not acceptable — no time dependence

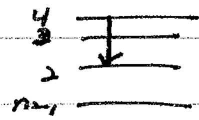
b) Not acceptable — $\psi(0)$ and $\psi(L)$ do not vanish.

c) Acceptable

d) Acceptable

e) Not acceptable — second term has wrong time dependence.

4. a) $E_\gamma = Z^2 E_R \left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$, $Z=2$ for He



Balmer-β : $n=2$, $n'=4$

$$E_\gamma = 4E_R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3}{4}E_R$$

Frequency $f = \frac{E_\gamma}{h} = \boxed{\frac{3}{4}\frac{E_R}{h}}$

b) $E_\gamma = 4E_R\left(\frac{1}{4} - \frac{1}{16}\right) = E_R\left(1 - \frac{1}{4}\right)$

$= E_R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

This is equivalent to the $n=2$ to $n=1$ transition of hydrogen, i.e. the $\boxed{\text{Lyman-}\alpha\text{ line.}}$

c) The force on the electron is $F = \frac{mv^2}{r} = \frac{ke^2}{r^2}$.

Hence, $\frac{1}{2}mv^2 = \frac{1}{2}\frac{ke^2}{r} = -\frac{1}{2}U$, where U = potential energy.

$$E = \frac{1}{2}mv^2 + U = -\frac{1}{2}mv^2$$

Since $E = -E_R$ in the ground state,

$\frac{1}{2}mv^2 = E_R$

$\Longrightarrow \boxed{v = \sqrt{\frac{2E_R}{m}}}$