

Phys 201 F'09 Exam 1 Solutions

$$1. \frac{\lambda_{\text{obs}}}{\lambda_{\text{src}}} = \left(\frac{1+\beta}{1-\beta} \right)^{1/2} = 1.01$$

$$\approx \left((1+\beta)(1+\beta) \right)^{1/2} = 1+\beta \quad \text{assuming } \beta \ll 1.$$

$\Rightarrow \beta \approx 0.01$ consistent with $\beta \ll 1$.

$$\boxed{v = 0.01c}$$

$$2. a) t_{\text{Phelps}} = \frac{1 \text{ c}\cdot\text{min}}{4c/5} = \boxed{\frac{5}{4} \text{ min}}$$

$$b) t_{\text{Thorpe}} = \frac{1/2 \text{ c}\cdot\text{min}}{2c/5} = \boxed{\frac{5}{4} \text{ min}}$$

c) In Phelps' frame, $t' = t/\gamma$.

$$\gamma = \left(1 - \frac{v_{\text{Phelps}}^2}{c^2} \right)^{-1/2} = \left(1 - \left(\frac{4}{5} \right)^2 \right)^{-1/2} = \frac{5}{3}$$

$$t' = \left(\frac{5}{4} \text{ min} \right) \cdot \frac{3}{5} = \frac{3}{4} \text{ min} = 45 \text{ sec}$$

Time on Phelps' watch $\boxed{12:00 \text{ and } 45 \text{ sec.}}$

$$d) u_y' = \frac{u_y}{\gamma} = \frac{2c/5}{5/3} = \boxed{\frac{6}{25} c}$$

e) In the $t' = \frac{3}{4} \text{ min}$, Thorpe travels a distance

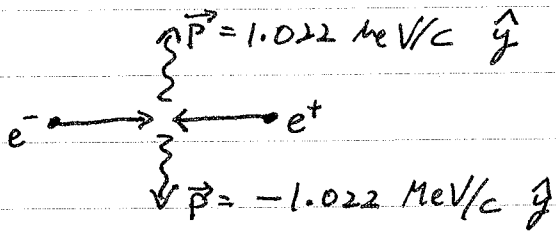
$$u_y' t' = \left(\frac{6}{25} c \right) \left(\frac{3}{4} \text{ min} \right) = \boxed{\frac{9}{50} \text{ c}\cdot\text{min}}$$

f) Extra distance Thorpe needs to travel

$$\Delta d = \frac{1}{2} \text{ c}\cdot\text{min} - \frac{9}{50} \text{ c}\cdot\text{min} = \frac{8}{25} \text{ c}\cdot\text{min}$$

During the head start Phelps is not swimming, so the extra time needed by Thorpe is $\frac{\Delta d}{u_y} = \frac{8/25 \text{ c}\cdot\text{min}}{2/5 c} = \frac{4}{5} \text{ min} = \boxed{48 \text{ sec}}$

3.



$$\begin{aligned} \vec{P}_{\text{tot}} &= 0 \\ &= m_e \vec{u}_e \gamma_e + m_{e^+} \vec{u}_{e^+} \gamma_{e^+} \\ m_{e^-} &= m_{e^+} \equiv m_e \rightarrow \vec{u}_{e^-} = -\vec{u}_{e^+} \end{aligned}$$

$$\begin{aligned} E_{\text{tot}} &= 2pc = 2.044 \text{ MeV} \\ &= 2m_e c^2 \gamma \\ &= 2(0.511 \text{ MeV}/c^2) c^2 \gamma \end{aligned}$$

$$\Rightarrow \gamma = \frac{2.044}{1.022} = 2$$

$$\left(1 - \frac{u^2}{c^2}\right)^{-1/2} = 2$$

$$1 - \frac{u^2}{c^2} = \frac{1}{4} \Rightarrow \boxed{u = \frac{\sqrt{3}}{2} c}$$