

Physics 201, Fall 2008

Final Exam

December 8, 2008 1:30-4:30 pm

Write your name on each page of paper submitted. You must show all work to receive credit, but only turn in work you would like graded. This exam is closed book. **You may use a calculator**, but no other electronic devices. There are four problems. Good luck!

Possibly Useful Relations:

Quantum mechanics:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x}) + V(\mathbf{x})\psi(\mathbf{x}) = E\psi(\mathbf{x})$$

$$i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{x},t) + V(\mathbf{x})\Psi(\mathbf{x},t)$$

$$E = \hbar\omega, \quad \mathbf{p} = \hbar\mathbf{k}$$

Hydrogen atom:

Wavefunctions $\psi_{nlm}(\mathbf{x})$

$$E_n = -E_R/n^2, \quad \mathbf{L}^2 = \hbar^2 l(l+1), \quad L_z = \hbar m$$

$$E_R = \frac{m_e(k_e^2)^2}{2\hbar^2} = 13.6 \text{ eV}$$

1-D particle in a box:

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}, \quad \psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

Relativity:

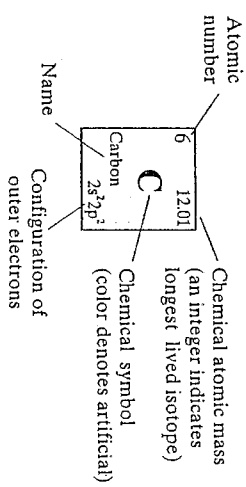
$$E^2 = \mathbf{p}^2 c^2 + m^2 c^4, \quad E = mc^2 \gamma, \quad \mathbf{p} = m\mathbf{u}\gamma$$

$$\gamma = \frac{1}{\sqrt{1 - \mathbf{u}^2/c^2}}$$

$$\Delta t' = \Delta t/\gamma, \quad l' = l\gamma$$

THE PERIODIC TABLE OF ELEMENTS

KEY



1 H Hydrogen 1s ¹	2 He Helium 1s ²	3 Li Lithium 2s ¹	4 Be Beryllium 2s ²	5 B Boron 2s ² 2p ¹	6 C Carbon 2s ² 2p ²	7 N Nitrogen 2s ² 2p ³	8 O Oxygen 2s ² 2p ⁴	9 F Fluorine 2s ² 2p ⁵	10 Ne Neon 2s ² 2p ⁶	11 Na Sodium 3s ¹	12 Mg Magnesium 3s ²	13 Al Aluminum 3s ² 3p ¹	14 Si Silicon 3s ² 3p ²	15 P Phosphorus 3s ² 3p ³	16 S Sulfur 3s ² 3p ⁴	17 Cl Chlorine 3s ² 3p ⁵	18 Ar Argon 3s ² 3p ⁶	19 K Potassium 4s ¹	20 Ca Calcium 4s ²	21 Sc Scandium 3d ¹ 4s ²	22 Ti Titanium 3d ² 4s ²	23 V Vanadium 3d ³ 4s ¹	24 Cr Chromium 3d ⁵ 4s ¹	25 Mn Manganese 3d ⁵ 4s ²	26 Fe Iron 3d ⁶ 4s ²	27 Co Cobalt 3d ⁷ 4s ²	28 Ni Nickel 3d ⁸ 4s ²	29 Cu Copper 3d ¹⁰ 4s ¹	30 Zn Zinc 3d ¹⁰ 4s ²	31 Ga Gallium 3d ¹⁰ 4s ² 4p ¹	32 Ge Germanium 3d ¹⁰ 4s ² 4p ²	33 As Arsenic 3d ¹⁰ 4s ² 4p ³	34 Se Selenium 3d ¹⁰ 4s ² 4p ⁴	35 Br Bromine 3d ¹⁰ 4s ² 4p ⁵	36 Kr Krypton 3d ¹⁰ 4s ² 4p ⁶	37 Rb Rubidium 5s ¹	38 Sr Strontium 5s ²	39 Y Yttrium 4d ¹ 5s ²	40 Zr Zirconium 4d ² 5s ²	41 Nb Niobium 4d ⁴ 5s ¹	42 Mo Molybdenum 4d ⁵ 5s ¹	43 Tc Technetium 4d ⁵ 5s ²	44 Ru Ruthenium 4d ⁷ 5s ¹	45 Rh Rhodium 4d ⁸ 5s ¹	46 Pd Palladium 4d ¹⁰	47 Ag Silver 4d ¹⁰ 5s ¹	48 Cd Cadmium 4d ¹⁰ 5s ²	49 In Indium 4d ¹⁰ 5s ² 5p ¹	50 Sn Tin 4d ¹⁰ 5s ² 5p ²	51 Sb Antimony 4d ¹⁰ 5s ² 5p ³	52 Te Tellurium 4d ¹⁰ 5s ² 5p ⁴	53 I Iodine 4d ¹⁰ 5s ² 5p ⁵	54 Xe Xenon 4d ¹⁰ 5s ² 5p ⁶	55 Cs Cesium 6s ¹	56 Ba Barium 6s ²	57-71 * LANTHANIDES † ACTINIDES	72 Hf Hafnium 4f ¹⁴ 5d ² 6s ²	73 Ta Tantalum 4f ¹⁴ 5d ³ 6s ²	74 W Tungsten 4f ¹⁴ 5d ⁴ 6s ²	75 Re Rhenium 4f ¹⁴ 5d ⁵ 6s ²	76 Os Osmium 4f ¹⁴ 5d ⁶ 6s ²	77 Ir Iridium 4f ¹⁴ 5d ⁷ 6s ²	78 Pt Platinum 4f ¹⁴ 5d ⁹ 6s ¹	79 Au Gold 4f ¹⁴ 5d ¹⁰ 6s ¹	80 Hg Mercury 4f ¹⁴ 5d ¹⁰ 6s ²	81 Tl Thallium 4f ¹⁴ 5d ¹⁰ 6s ² 6p ¹	82 Pb Lead 4f ¹⁴ 5d ¹⁰ 6s ² 6p ²	83 Bi Bismuth 4f ¹⁴ 5d ¹⁰ 6s ² 6p ³	84 Po Polonium 4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁴	85 At Astatine 4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁵	86 Rn Radon 4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁶	87 Fr Francium 7s ¹	88 Ra Radium 7s ²	89-103 † ACTINIDES	104 Rf Rutherfordium 5f ¹⁴ 6d ² 7s ²	105 Db Dubnium 5f ¹⁴ 6d ³ 7s ²	106 Sg Seaborgium 5f ¹⁴ 6d ⁴ 7s ²	107 Bh Bohrium 5f ¹⁴ 6d ⁵ 7s ²	108 Hs Hassium 5f ¹⁴ 6d ⁶ 7s ²	109 Mt Meitnerium 5f ¹⁴ 6d ⁷ 7s ²	110 Ds Darmstadtium 5f ¹⁴ 6d ⁸ 7s ²	111 Rg Roentgenium 5f ¹⁴ 6d ⁹ 7s ²	112 Cn Copernicium 5f ¹⁴ 6d ¹⁰ 7s ²	113 Nh Nihonium 5f ¹⁴ 6d ¹⁰ 7s ² 7p ¹	114 Fl Flerovium 5f ¹⁴ 6d ¹⁰ 7s ² 7p ²	115 Mc Moscovium 5f ¹⁴ 6d ¹⁰ 7s ² 7p ³	116 Lv Livermorium 5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁴	117 Ts Tennessine 5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁵	118 Og Oganesson 5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁶
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LANTHANIDES

57 La Lanthanum 5d ¹ 6s ²	58 Ce Cerium 4f ¹ 5d ¹ 6s ²	59 Pr Praseodymium 4f ² 6s ²	60 Nd Neodymium 4f ³ 6s ²	61 Pm Promethium 4f ⁵ 6s ²	62 Sm Samarium 4f ⁶ 6s ²	63 Eu Europium 4f ⁷ 6s ²	64 Gd Gadolinium 4f ⁷ 5d ¹ 6s ²	65 Tb Terbium 4f ⁹ 6s ²	66 Dy Dysprosium 4f ¹⁰ 6s ²	67 Ho Holmium 4f ¹¹ 6s ²	68 Er Erbium 4f ¹² 6s ²	69 Tm Thulium 4f ¹³ 6s ²	70 Yb Ytterbium 4f ¹⁴ 6s ²	71 Lu Lutetium 4f ¹⁴ 5d ¹ 6s ²
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ACTINIDES

89 Ac Actinium 6d ¹ 7s ²	90 Th Thorium 6d ² 7s ²	91 Pa Protactinium 5f ² 6d ¹ 7s ²	92 U Uranium 5f ³ 6d ¹ 7s ²	93 Np Neptunium 5f ⁴ 6d ¹ 7s ²	94 Pu Plutonium 5f ⁶ 7s ²	95 Am Americium 5f ⁷ 7s ²	96 Cm Curium 5f ⁸ 6d ¹ 7s ²	97 Bk Berkelium 5f ⁹ 7s ²	98 Cf Californium 5f ¹⁰ 7s ²	99 Es Einsteinium 5f ¹¹ 7s ²	100 Fm Fermium 5f ¹² 7s ²	101 Md Mendelevium 5f ¹³ 7s ²	102 No Nobelium 5f ¹⁴ 7s ²	103 Lr Lawrencium 5f ¹⁴ 6d ¹ 7s ²
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From Taylor, Zafrales + Dubson

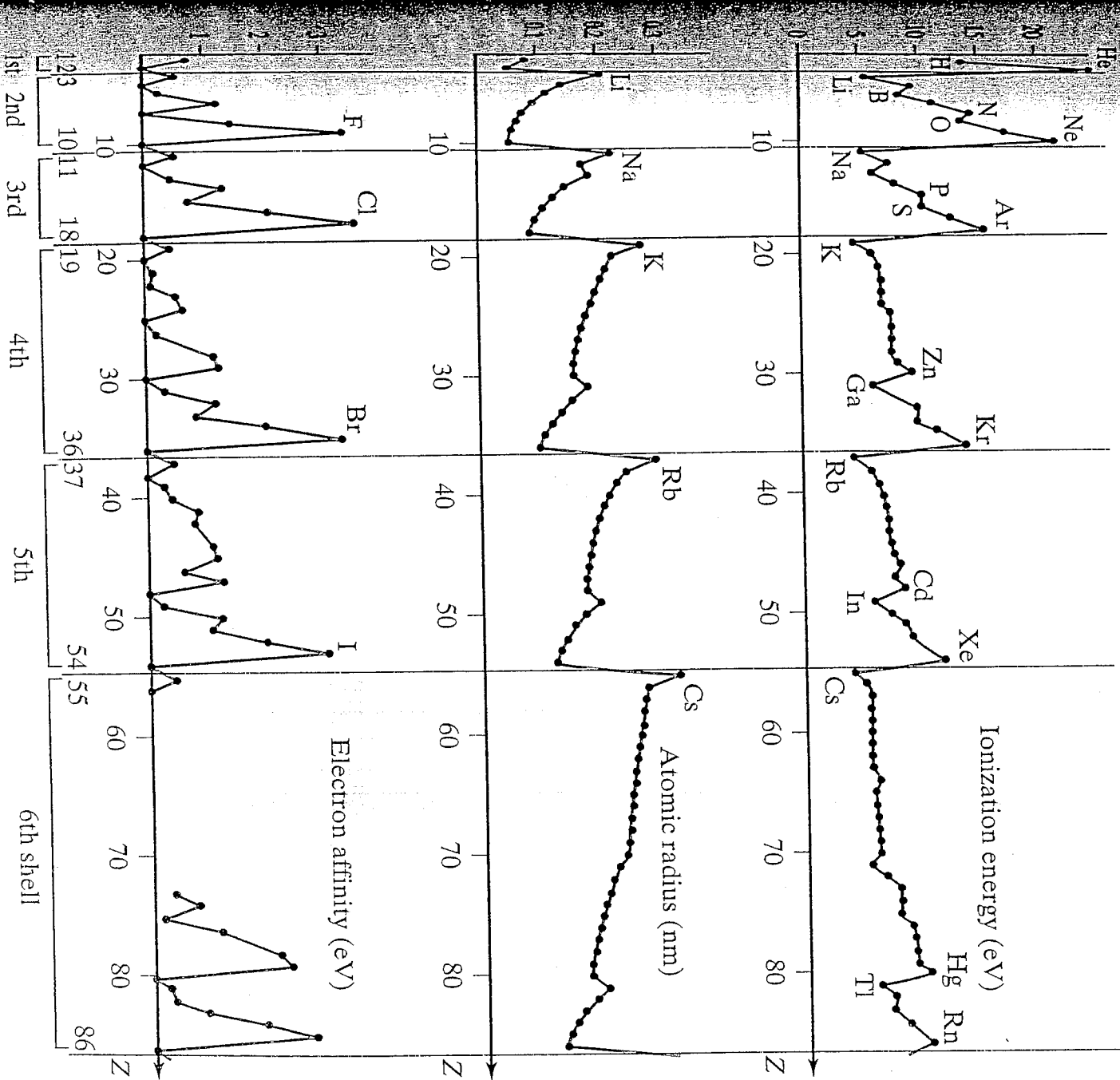


FIGURE 10.11

Ionization energy, atomic radius, and electron affinity as functions of atomic number Z . The vertical lines separate complete shells. The electron affinities for elements 57 through 72 have not been measured.

1. *Atoms and Molecules*

You may want to refer to the periodic table for this problem.

a) (5 points) The first ionization energy of an atom is the energy required to remove one electron. For helium, this is 24.6 eV. The second ionization energy is the *additional* energy required to remove a second electron. Calculate the second ionization energy of helium.

b) (5 points) What is the total atomic binding energy of helium, *i.e.* the energy required to take the atom apart into two separated electrons and a nucleus?

c) (5 points) Assuming that the following pairs of elements combine ionically, write down the chemical formulae of the resulting molecules: Ca and S, Li and O, Na and P.

d) (5 points) Assuming that the following pairs of elements combine covalently, write down the formulae of the resulting molecules: Cl and F, Br and Cl, I and Cl.

2. Solutions to the Schrödinger Equation

a) (5 points) Suppose at time $t = 0$ the electron in a hydrogen atom has wavefunction

$$\psi(\mathbf{x}) = \frac{1}{\sqrt{10}} (\psi_{100}(\mathbf{x}) + 3\psi_{211}(\mathbf{x})),$$

where the energy eigenfunctions $\psi_{nlm}(\mathbf{x})$ are properly normalized. What is the probability that the energy of the electron would be measured to be $E = -E_R/4$?

b) (5 points) What is the wavefunction for the electron of part (a) at time t ?

c) (15 points) Consider the three-dimensional particle in a box of dimensions $L \times L \times L$. The potential energy function vanishes if $0 < x < L$ and $0 < y < L$ and $0 < z < L$, but is infinite otherwise.

Assume a solution to the time-independent Schrödinger equation inside the box of the form $\psi(x, y, z) = \psi_x(x)\psi_y(y)\psi_z(z)$.

Using separation of variables, find separate differential equations for $\psi_x(x)$, $\psi_y(y)$ and $\psi_z(z)$.

d) (10 points) The wavefunction must vanish at the boundaries of the box, so

$$\psi_x(0) = \psi_x(L) = 0$$

$$\psi_y(0) = \psi_y(L) = 0$$

$$\psi_z(0) = \psi_z(L) = 0.$$

What are the nonvanishing solutions for $\psi(x, y, z)$, and what are their corresponding energies? (Don't worry about the normalization.)

e) (5 points) What are the degeneracies of the ground state and first excited state of this system?

f) (5 points) Assume five electrons are in the box considered above. Ignoring the electric repulsion of the electrons, what is the total energy of the ground state of the system of five electrons? (*Hint:* It may help to compare this system to a multielectron atom.)

3. Special Relativity

a) (5 points) Explain how relativity predicts that massless particles travel at the speed of light.

b) (20 points) A particle of unknown mass M decays into two particles of known masses $m_1 = 2\sqrt{5} \text{ GeV}/c^2$ and $m_2 = 4 \text{ GeV}/c^2$, whose momenta are measured to be $\mathbf{p}_1 = 4 \text{ GeV}/c$ along the positive y axis, and $\mathbf{p}_2 = 3 \text{ GeV}/c$ along the positive x axis, respectively.

Find the unknown mass M and the particle's speed before it decayed.

c) (10 points) Rod B is moving relative to rod A with speed $3c/5$ as in the figure below. In rod A's rest frame, both rods are measured to have length L . What is the ratio of the length of rod A to the rest length of rod B as measured in rod B's rest frame?

