Phys 201 F'08 Final Exam Solutions

1. a) After one electron is removed from the He atom, what is left is an He^+ ion. Its ionization energy is $E_{He^+} = \boxed{+4E_R} = \boxed{+54.4\ eV}$

Z^2, with $Z=2$

b) Binding energy $= 24.6\ eV + 54.4\ eV = \boxed{79.0\ eV}$

c) Ca $4s^2$, S $3s^2 3p^4$

closed shell +2 closed shell −2 $\rightarrow \boxed{CaS}$

Li $2s^1$, O $2s^2 2p^4$

closed +1 closed −2 $\rightarrow \boxed{Li_2O}$

Na $3s^1$, P $3s^2 3p^3$

closed +1 closed −3 $\rightarrow \boxed{Na_3P}$

d) Cl $3s^2 3p^5$ $\underset{3s}{\uparrow\downarrow}$ $\underset{3p}{\uparrow\downarrow\ \uparrow\downarrow\ \uparrow}$ valence = 1

F $2s^2 2p^5$ $\underset{2s}{\uparrow\downarrow}$ $\uparrow\downarrow\ \uparrow\downarrow\ \uparrow$ valence = 1

$\Big\}\ \boxed{ClF}$

Br $4s^2 4p^5$ valence = 1 $\rightarrow \boxed{BrCl}$

I $5s^2 5p^5$ valence = 1 $\rightarrow \boxed{ICl}$

2. a) Prob $\left(E = -E_R/4\right) = \frac{3^2}{10} = \boxed{\frac{9}{10}}$

b) $\Psi(\vec{x},t) = \frac{1}{\sqrt{10}}\left(\psi_{100}(\vec{x})\, e^{-iE_1 t/\hbar} + 3\,\psi_{211}(\vec{x})\, e^{-iE_2 t/\hbar}\right)$

where $E_1 = -E_R$, $E_2 = -\frac{E_R}{4}$

c) $-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = E\psi$

$\psi(x,y,z) = \psi_x(x)\,\psi_y(y)\,\psi_z(z)$

$$-\frac{\hbar^2}{2m}\left(\underbrace{\frac{\psi_x''(x)}{\psi_x(x)}}_{\text{fn. of } x} + \underbrace{\frac{\psi_y''(y)}{\psi_y(y)}}_{\text{fn. of } y} + \underbrace{\frac{\psi_z''(z)}{\psi_z(z)}}_{\text{fn. of } z}\right) = E$$

← independent constants

$$\boxed{-\frac{\hbar^2}{2m}\frac{\psi_x''(x)}{\psi_x(x)} = E_x, \quad -\frac{\hbar^2}{2m}\frac{\psi_y''(y)}{\psi_y(y)} = E_y, \quad -\frac{\hbar^2}{2m}\frac{\psi_z''(z)}{\psi_z(z)} = E_z}$$

with $E_x + E_y + E_z = E$

d) $\psi_x(x) \propto \sin\left(\frac{n_x \pi x}{L}\right)$, $\psi_y(y) \propto \sin\left(\frac{n_y \pi y}{L}\right)$, $\psi_z(z) \propto \sin\left(\frac{n_z \pi z}{L}\right)$

$E_{n_x n_y n_z} = \frac{\hbar^2\pi^2}{2mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$; n_x, n_y, n_z integers >0.

e) Ground state: $n_x = n_y = n_z = 1$ → <u>nondegenerate</u>

1^{st} excited state: $(n_x, n_y, n_z) = (2,1,1)$ or $(1,2,1)$ or $(1,1,2)$

→ degeneracy = <u>3</u>

f) 5 electrons: By Pauli exclusion principle, 2 in ground state w/ $E = \frac{3\hbar^2\pi^2}{2mL^2}$, 3 in 1^{st} excited states w/ $E = \frac{3\hbar^2\pi^2}{mL^2}$

$\longrightarrow E = 2\left(\frac{3}{2}\frac{\hbar^2\pi^2}{mL^2}\right) + 3\left(\frac{3\hbar^2\pi^2}{mL^2}\right) = \boxed{\frac{12\hbar^2\pi^2}{mL^2}}$

3. a) $u = \frac{pc^2}{E} = \frac{pc^2}{\sqrt{p^2c^2+m^2c^4}} = c$ if $m=0$.

b) $\vec{p}_M = \vec{p}_1 + \vec{p}_2 = (3\hat{x} + 4\hat{y})$ GeV/c

$|\vec{p}_M| = 5$ GeV/c $\equiv p_M$

$E_M = (p_1^2c^2 + m_1^2c^4)^{1/2} + (p_2^2c^2 + m_2^2c^4)^{1/2}$

$= (16+20)^{1/2}$ GeV $+ (9+16)^{1/2}$ GeV

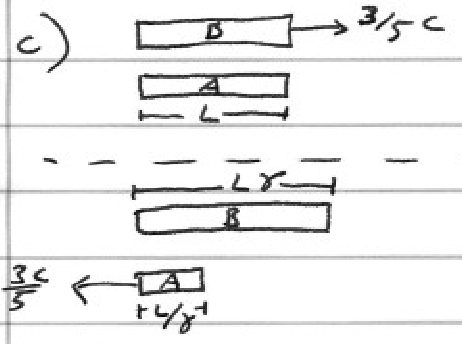
$=$ 6 GeV + 5 GeV = 11 GeV

$p_M^2c^2 + M^2c^4 = 25\,(\text{GeV})^2 + M^2c^4 = 121\,(\text{GeV})^2$

$M = \sqrt{96}$ GeV/c^2 $= \boxed{4\sqrt{6}\ \text{GeV}/c^2}$

$u_M = \frac{p_M c^2}{E_M} = \frac{5\ \text{GeV}\cdot c}{11\ \text{GeV}} = \boxed{\frac{5}{11}c}$

c)



Rod A rest frame

Rod B rest frame

$\frac{\text{Length of A}}{\text{Length of B}} = \frac{L/\gamma}{L\gamma} = \frac{1}{\gamma^2} = 1 - \frac{9}{25} = \boxed{\frac{16}{25}}$