1. \(P, C\) and \(T\)

a) Determine the action of \(P\), \(C\) and \(T\) on the fermion bilinears \(\bar{\psi}\psi\), \(i\bar{\psi}\gamma^5\psi\), \(\bar{\psi}\gamma^\mu\psi\), \(\bar{\psi}\gamma^\mu\gamma^5\psi\), and \(i\bar{\psi}[\gamma^\mu, \gamma^\nu]\psi\).

b) Convince yourself that any Lorentz invariant formed by fermion bilinears and/or space-time derivatives is invariant under the combined transformation CPT.

c) Convince yourself that QED is invariant under \(C\), \(P\), and \(T\) independently. How does the electromagnetic field \(A_\mu\) transform?

d) What are the discrete symmetries of the following Lagrangians? (Write the transformations of the fields which leave the action invariant.)

\[
\mathcal{L} = \bar{\psi}(i\partial^\mu - m)\psi + \frac{1}{2} (\partial_\mu \phi)^2 + g \bar{\psi}\psi \phi
\]

\[
\mathcal{L} = \bar{\psi}(i\partial^\mu - m)\psi + \frac{1}{2} (\partial_\mu \phi)^2 + ig \bar{\psi}\gamma^5\psi \phi
\]

\[
\mathcal{L} = \bar{\psi}(i\partial^\mu - m - eA^\mu)\psi - \frac{1}{4} F^\mu_{\nu} F^{\mu\nu} + ig \bar{\psi}[\gamma^\mu, \gamma^\nu] \psi F^\mu_{\nu}
\]