

Print out this exam (two pages), and turn it in with answers, plus your work sheets. There are 3 problems.

Open text (Jackson only), open notes (this class only). You may—if necessary—open a fresh Mathematica notebook or equivalent and use it, but you may only look at that one notebook and the exam on your or any other computer while taking the exam.

Name: \_\_\_\_\_

1. [35 points total] A beam of light, traveling in the  $z$ -direction, passes through four filters, labeled 0 through 3. "Filter" 0 passes all the light; filter 1 passes only light plane polarized in the  $x$  direction; filter 2 passes only light plane polarized at  $45^\circ$  between the  $x$  and  $y$  axes; and filter 3 passes only right-handed polarized light.

You measure that filter 1 passes 80% as much intensity as filter 0; filter 2 passes 84.6% as much intensity as filter 0; and filter 3 passes 37.5% as much intensity as filter 0.

a) Find the polarization of the beam. Specifically, writing the electric field of the initial beam as

$$\vec{E} = (E_1 \hat{x} + E_2 \hat{y}) e^{i(kz - \omega t)} = (a_1 \hat{x} + a_2 e^{i\phi} \hat{y}) e^{i(kz - \omega t)},$$

with  $a_1$  and  $a_2$  real and positive, and  $\phi$  real, find  $a_2/a_1$  and  $\phi$ .

b) The full intensity of the beam is 45 Watts/m<sup>2</sup>. Find  $a_1$  in appropriate units.

Answers:

a)  $a_2/a_1 =$  \_\_\_\_\_

$\phi =$  \_\_\_\_\_

b)  $a_1 =$  \_\_\_\_\_

2. [30 points total] Another beam of light, also traveling in the  $z$ -direction with

$$\vec{E} = (a_1 \hat{x} + a_2 e^{i\phi} \hat{y}) e^{i(kz - \omega t)},$$

has equal  $x$  and  $y$  amplitudes  $a_1 = a_2 = a$ .

a) The polarization vector will, as time passes, trace out an ellipse in the  $x$ - $y$  plane (take  $z = 0$ ) with its major axis at an angle  $\theta$  to the  $x$  axis. What is  $\theta$ ?

b) Sketch the polarization ellipse for the cases  $\phi = 60^\circ$  and  $\phi = 30^\circ$ . (You can put them on the same sketch; label which is which.)

Answers:

a)  $\theta =$  \_\_\_\_\_

b) I will find your sketch on your worksheets (or on the back of this sheet).

**3.** [35 points total] A particle of charge  $q$  travels in a straight line at speed  $v \neq 0$ , and passes at distance  $a$  at closest approach to a certain point. Work out the (retarded) electromagnetic potentials in Lorenz gauge at this point at the time of closest approach.

For definiteness, say that the charged particle is moving along the  $z$  axis and passes through the origin at time zero, and that the point in question is on the  $x$  axis, distance  $a$  from the origin.

- a) Show that only one point along the charged particle's path contributes to the desired potentials at time  $t = 0$ . Do so by finding that point along the path.
- b) Find the scalar potential at the point in question,  $\Phi = \Phi(x = a, y = 0, z = 0, t = 0)$ , in terms of  $q, a, v$ , and standard electromagnetic constants.
- c) Find the vector potential at the same point.

Answers:

a)  $x = 0, \quad y = 0, \quad z = \underline{\hspace{4cm}},$

b)  $\Phi = \underline{\hspace{4cm}},$

c)  $\vec{A} = \underline{\hspace{4cm}}.$