Physics 722 — Quantum Field Theory

Homework 5 — Due 27 April 2017

1. Show that the f_{abc} are totally antisymmetric, when all generators are normalized the same way. (The f_{abc} are structure constants of a Lie algebra defined from commutation relations of the generators,

$$[T_a, T_b] = i f_{abc} T_c \,,$$

and that all generators be normalized the same way means

$$\mathrm{Tr}\,(T_a T_b) = C \delta_{ab} \,,$$

where C is the same for all a.)

2. From $F_{\mu\nu}$ given as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu, A_\nu]$$

and the gauge transformation property of the A_μ field,

$$A_\mu \longrightarrow A'_\mu = U(x) \left(A_\mu + \frac{i}{g} \partial_\mu \right) U^\dagger(x),$$

show that $F_{\mu\nu}$ gauge transforms as

$$F_{\mu\nu} \longrightarrow F'_{\mu\nu} = U(x) F_{\mu\nu} U^\dagger(x).$$

3. From the matrices for the generators of the fundamental representation of $SU(3)$ given in class, or in Problem 15.1 of Peskin and Schroeder, calculate the structure constants f_{147}, f_{471}, and f_{714} and verify that they have the expected symmetry or antisymmetry relations.

4. Write down—or at least think clearly about—the generators of the fundamental representation of $SO(4)$. How many $SO(4)$ generators are there? Calculate the Casimir operator for this case, and show that it fits expectations.

5. Peskin and Schroeder problem 16.1,

or

if you prefer a possibly simpler alternative,
Calculate the gauge boson propagator that follows from the action

$$S = \int d^4x \, \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \lambda (n \cdot A)^2 \right),$$

where n is unit vector in z-direction, where $F_{\mu\nu}$ has only the lowest order terms $\partial_\mu A_\nu - \partial_\nu A_\mu$, and the limit $\lambda \to \infty$ is to be taken.