

1. (a) From the recursion formulas given in Jackson [Eq. (9.90)], show that the spherical Bessel functions satisfy,

$$j_{\ell-1}(x) = \frac{1}{x^{\ell+1}} \frac{d}{dx} \left(x^{\ell+1} j_{\ell}(x) \right).$$

(b) Also show that

$$j_{\ell+1}(x) = -x^a \frac{d}{dx} \left(x^{-a} j_{\ell}(x) \right),$$

where a is some integer you should find in terms of ℓ .

(c) Explicitly show that the latter formula with $j_{\ell}(x) = j_0(x) = (\sin(x))/x$ gives the correct $j_1(x)$ (quoted in Jackson).

2. Jackson 9.2

3. Jackson 9.3

4. Jackson 10.1