**Physics 611** **Homework #3** **handed out 13 October 2016**
**due 20 October 2016**

**1.** a) Find the surface resistance for a silver surface working at room temperature with 20 cm wavelength microwaves. (You will have to look up the conductance or resistivity of silver.)

b) What is the actual scale depth of the surface current for the above conditions?

c) If the tangential electric field at the surface is 4,000 V/m, how much resistive heating is there in each square meter of silver surface?

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a)

$$\rho_{\text{surf}} = \frac{1}{2\sigma\delta} = \sqrt{\frac{\mu_c\omega}{8\sigma}}$$

$$\sigma = \frac{1}{\rho} = 6.3 \times 10^7\ \Omega^{-1}\text{m}^{-1}$$

$$\omega = \frac{2\pi c}{\lambda} = 9.4 \times 10^9\ \text{s}^{-1}$$

$$\mu_c \approx \mu_0 = 4\pi \times 10^{-7}\ \text{N/A}^2$$

$$\therefore\ \rho_{\text{surf}} = 4.8 \times 10^{-3}\ \Omega \tag{1}$$

b)

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} = 1.6 \times 10^{-6}\ \text{m} = 1.6\ \mu\text{m} \tag{2}$$

c) From the notes, the electric field at the surface obeys

$$E_\parallel = \frac{1-i}{\sigma\delta}\hat{n} \times H_\parallel\,,$$

$$|E_\parallel| = \frac{\sqrt{2}}{\sigma\delta}|H_\parallel| = \frac{\sqrt{2}}{\sigma\delta}|K| \tag{3}$$

Then

$$\frac{P}{a} = \frac{1}{2\sigma\delta}|K|^2 = \frac{\sigma\delta}{4}|E_\parallel|^2 = 4.1 \times 10^8\ \frac{\text{W}}{\text{m}^2} \tag{4}$$

(Good reason to go superconducting.)

**2.** Jackson problem 8.1.

Jackson 8.1

a) Let $\vec{f}_V$ = force/volume

$\vec{f}_A$ = force/area ~~(this is just $\vec{f}$ in Jackson)~~

Lorentz force law: $\vec{f}_V = \rho\vec{E} + \vec{J}\times\vec{B}$

There are no free charges, so only magnetic term matters.

Within conductor
$$H_c = H_{\parallel}\, e^{-(1-i)\xi/\delta}$$
$$E_c = \frac{1}{\sigma\delta}(1-i)(\hat{n}\times H_{\parallel})e^{-(1-i)\xi/\delta}$$
$$J = \sigma E_c = \frac{1-i}{\delta}(\hat{n}\times H_{\parallel})e^{-(1-i)\xi/\delta}$$

$$\langle f_V\rangle = \frac{1}{2}\operatorname{Re} J\times B^* = \frac{\mu}{2}\operatorname{Re}\, \sigma E_c\times H_c^*$$
$$= \frac{\mu}{2}\operatorname{Re}\left\{\frac{1-i}{\delta}(\hat{n}\times H_{\parallel})\times H_{\parallel}^*\, e^{-(1-i)\xi/\delta}e^{-(1+i)\xi/\delta}\right\}$$
$$= \frac{\mu}{2\delta}\underbrace{(\hat{n}\times H_{\parallel})\times H_{\parallel}^*}_{H_{\parallel}(\hat{n}\cdot H_{\parallel}) - \hat{n}|H_{\parallel}|^2}\, e^{-2\xi/\delta}$$

(should do this before taking real part)

$$\langle f_V\rangle = -\hat{n}\,\frac{\mu}{2\delta}|H_{\parallel}|^2 e^{-2\xi/\delta}$$
$$\underbrace{\langle f_A\rangle \equiv f}_{\text{in Jackson}} = -\hat{n}\,\frac{\mu}{2\delta}|H_{\parallel}|^2\int_0^\infty e^{-2\xi/\delta}\,d\xi$$

$$\boxed{\langle f_A\rangle = f = -\hat{n}\,\frac{\mu_c}{4}|H_{\parallel}|^2}$$

b) I don't see anything.

Electric forces don't contribute.

(c) Makes sense to me only if the meaning of the symbols is defined.

Monochromatic: $H_c(\vec{x},t) = H_{\parallel} e^{-(1-i)\xi/\delta} e^{-i\omega t}$

Here $H_{\parallel}$ ~~has~~ is a constant and the time dependence is in the ~~e~~ $e^{-i\omega t}$ factor.

Superposition case:

$$H_c(\vec{x},t) = \sum_i H_{\parallel}(\omega_i) e^{-(1-i)\xi/\delta_i} e^{-i\omega_i t}$$

Define $H_{\parallel}(t)$ as something with the time variation at the surface.

$$H_{\parallel}(t) = H_c(0,t) = \sum_i H_{\parallel}(\omega_i) e^{-i\omega_i t}$$

Note that

$$\langle |H_{\parallel}|^2 \rangle = \langle |\mathrm{Re}\{\sum_i H_{\parallel}(\omega_i) e^{-i\omega_i t}\}|^2 \rangle$$
$$= \sum_i \langle |\mathrm{Re}\, H_{\parallel}(\omega_i) e^{-i\omega_i t}|^2 \rangle$$
$$= \frac{1}{2} \sum_i |H_{\parallel}(\omega_i)|^2$$

The force:

$$E_c(\vec{x},t) = \sum_i \frac{1-i}{\sigma\delta_i} \hat{n} \times H_{\parallel}(\omega_i) e^{-(1-i)\xi/\delta_i} e^{-i\omega_i t}$$
$$\equiv \sum_i E_i(\omega_i) e^{-(1-i)\xi/\delta_i - i\omega_i t}$$

$$\cancel{\langle f_A \rangle = \langle \mathrm{Re}\{\sigma E_c \times H_c\}\rangle}$$
$$\cancel{= \sum_i \langle \mathrm{Re}\{\sigma E_c(\omega_i) e^{-(1-i)\xi/\delta_i} e^{-i\omega_i t} \times H_{\parallel}(\omega_i) e^{-i\omega_i}}$$

$$\langle f_A \rangle = \mu \langle \mathrm{Re}(\sigma E_c) \times \mathrm{Re}\, H_c \rangle$$
$$= \frac{\mu}{2} \sum_i \mathrm{Re}\{\sigma E_c(\omega_i) e^{-(1-i)\xi/\delta_i} \cdot H^*_{\parallel}(\omega_i) e^{-(1+i)\xi/\delta_i}\}$$
$$= \frac{\mu}{2} \sum_i \mathrm{Re}\, \frac{1-i}{\delta_i} \underbrace{(\hat{n} \times H_{\parallel}) \times H^*_{\parallel}(\omega_i)}_{-\hat{n}|H_{\parallel}(\omega_i)|^2} e^{-2\xi/\delta_i}$$
$$= -\hat{n} \frac{\mu}{2\delta} \sum_i |H_{\parallel}(\omega_i)|^2 e^{-2\xi/\delta_i}$$

$$\therefore \boxed{\langle f_A \rangle = -\hat{n} \frac{\mu}{4} \sum_i |H_{\parallel}(\omega_i)|^2 = -\hat{n} \frac{\mu}{2} \langle |H_{\parallel}|^2 \rangle},$$

the result found in the statement of the problem.