Physics 722 Quantum Field Theory

Homework 2 Due 23 February 2017

1. For Grassmann variables ξ and θ, show that one has a Fourier transform pair

$$g(\xi) = \int d\theta\, e^{i\xi\theta}\, f(\theta)\,, \tag{1}$$

$$f(\theta) = i \int d\xi\, e^{-i\xi\theta}\, g(\xi)\,. \tag{2}$$

I.e., using the first equation to define $g(\xi)$, show that the second equation is true.

2. Calculate the superficial degree of divergence in 4D for Yukawa theory, which is a theory of interacting fermions and scalars,

$$\mathcal{L} = \mathcal{L}_{0\psi} + \mathcal{L}_{0\phi} - g\bar{\psi}\psi\phi\,.$$

Look up the meaning of renormalizable, superrenormalizable, and non-renormalizable, and state which of these Yukawa theory is in 4D.

3. Calculate the superficial degree of divergence for scalar electrodynamics (charged pions and photons) in d dimensions, *i.e.,* where the integrations are d^dk and the propagators have the same powers of momentum as in 4D. Show that in 4D that the superficial degree of divergence of a Feynman diagram depends only on the numbers of external legs.

(The Lagrangian is

$$\begin{aligned}\mathcal{L} &= [(\partial - ieA)_\mu\phi]^* (\partial - ieA)^\mu\phi - m^2\phi^*\phi \\ &= \mathcal{L}_0 - ie\phi^* \left(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu\right)\phi A^\mu + e^2\phi^*\phi A_\mu A^\mu\ ,\end{aligned}$$

where the interaction terms give Feynman rules $-ie(p+p')_\mu$ and $2ie^2 g_{\mu\nu}$, respectively.)

My answer for the last problem (in four dimensions) is $D = 4 - N_\pi - N_\gamma$.