Physics 611 **Homework #1** **handed out 01 September 2016**

1. In Heaviside-Lorentz units the force between two point charges can be written as (in magnitude)

$$F = \frac{1}{4\pi}\frac{|q_1 q_2|}{r^2},$$

and the Lorentz force is

$$\vec{F} = q\vec{E} + q\frac{\vec{v}}{c}\times\vec{B}.$$

a) Write Maxwell's equations in Heaviside-Lorentz units.
b) The energy density in the electric and magnetic fields in SI units is

$$u = \frac{\epsilon_0}{2}\vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2.$$

Rewrite the energy density formula in gaussian and in Heaviside-Lorentz units.
c) Work out what magnetic field in gaussian units corresponds to a magnetic field of 1 Tesla in SI units.

2. Show that for a general but suitably differentiable function $z = z(x, y)$ that

$$\left.\frac{\partial x}{\partial y}\right|_z \left.\frac{\partial y}{\partial z}\right|_x \left.\frac{\partial z}{\partial x}\right|_y = -1.$$

3. ϵ exercises. Tensor ϵ_{ijk} (where each of i, j, k has allowed values $1, 2, 3$) is a totally antisymmetric tensor normalized by $\epsilon_{123} = 1$.
Notice that the ϵ-symbol is useful for writing cross products: $(\vec{A}\times\vec{B})_i = \epsilon_{ijk}A_jB_k \equiv \sum_{jk}\epsilon_{ijk}A_jB_k$.
a) Convince yourself that

$$\sum_i \epsilon_{ijk}\epsilon_{ilm} \equiv \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}\,.$$

b) Show that

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = -\nabla^2\vec{B} + \vec{\nabla}\left(\vec{\nabla}\cdot\vec{B}\right)\,.$$

c) Show that

$$(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = (\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})\,.$$

4. Jackson problem 6.1.

5. Jackson problem 6.11. (If you are uncertain about the "solar wind,", you may omit the response to the question in the last sentence of the problem.)