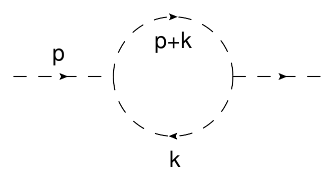
**Quantum Field Theory** **Physics 721**

Final Exam Three hours 02 May 2012

**1.** [35 points total] Consider $\phi^3$ theory, which has one scalar field $\phi$ (mass $m$) and an interaction Lagrangian with a real coupling parameter $g$,

$$\mathscr{L}_{int} = -\frac{1}{3!} g \phi^3 .$$

a) Calculate the superficial degree of divergence for a Feynman graph in this theory, in terms of the number of its external lines and the number of its vertices.
b) Draw and state the superficial degree of divergence of all divergent graphs in this theory, including the one shown below.




c) Calculate the graph shown. Carry the calculation far enough to combine the Feynman propagators, shift the integration variable, and go to Euclidean space so that the integrand is spherically symmetric. Stop (for this exam) just before actually doing the loop integral. What is the actual degree of divergence of this graph?

**2.** [35 points total] If a muon could decay into an electron and a photon, the interaction Hamiltonian might look like

$$\mathscr{H}_{int} = g\left[\bar{\psi}_\mu \gamma^\alpha \psi_e + \bar{\psi}_e \gamma^\alpha \psi_\mu\right] A_\alpha \ ,$$

where $\alpha$ is a Lorentz index, $\psi_\mu$ is the muon field, $\psi_e$ is the electron field, and $A_\alpha$ is the photon field.
Calculate the total $\mu \to e + \gamma$ decay rate in terms of the constant $g$ and the muon mass, using this interaction Hamiltonian. Neglect the electron mass. Steps along the way include drawing the Feynman diagram and writing down the decay amplitude $\mathscr{M}$.
(Note for the purists: The above does not satisfy current conservation, but one can fix this with extra terms that make the electron-muon matrix element $\bar{u}_e\left(\gamma^\alpha - q^\alpha \not{q}/q^2\right) u_\mu$, where $q$ is the photon momentum. If you have extra time, you may consider why the extra term does not affect the result.)

**3.** [30 points total] Here is the Polyakov string action with an extra term,

$$S_P = \frac{1}{4\pi\alpha'} \int d\tau \int_0^\pi d\sigma \left\{ \frac{\partial x_\mu}{\partial \tau}\frac{\partial x^\mu}{\partial \tau} - \frac{\partial x_\mu}{\partial \sigma}\frac{\partial x^\mu}{\partial \sigma} + 2F_{\mu\lambda}\frac{\partial x^\mu}{\partial \tau}\frac{\partial x^\lambda}{\partial \sigma} \right\} ,$$

where $F_{\mu\lambda}$ is an antisymmetric array of constants.
a) Show that the equation of motion that comes from the modified Polyakov action, or Lagrangian, is not changed by the extra term.
b) Find the boundary conditions that one gets for the ends of the string. (These may differ from the $\partial x^\mu/\partial\sigma = 0$ at the endpoints that you recall from the unmodified case.)

Side note: Even with the extra term, we can solve exactly the string Lagrangian, by finding the commutation relations of the Fourier coefficients of the $x_\mu$. A consequence is that the equal-$\tau$ commutators of the coordinates become nonzero, with $[x_\mu, x_\nu] \propto F_{\mu\nu}$ for small $F_{\mu\nu}$.