Classical E&M II **Physics 611**

Final Exam 3 hours 10 or 16 December 2015, 9:00 am

Turn in this question sheet with answers, plus your work sheets. There are 3 problems.

Open text (Jackson only), open notes (this class only), open old homework (this class only).

Name: ______________________________

1. [30 points total]
a) With the Born approximation, compute the unpolarized differential cross section for the scattering electromagnetic radiation (with wave number k) off a uniform dielectric sphere of radius a and relative dielectric constant $\epsilon_r = \epsilon/\epsilon_0 \approx 1$. [Note the word "unpolarized." Any polarization sums or averages must be done.]

b) Show that in the limit $ka \ll 1$ the differential cross section reduces to the small sphere result found in your notebooks, and which may be expressed as,

$$\frac{d\sigma}{d\Omega} = \frac{1}{18} k^4 a^6 |1 - \epsilon_r|^2 (1 + \cos^2\theta).$$

c) Express the $q = |\vec{q}|$ that may have appeared in part (a) in terms of k and the angle θ between the incoming and outgoing waves. Recall $\vec{q} = k(\hat{n}_0 - \hat{n})$ and where $\hat{n}_0$ is the direction of the incoming wave and $\hat{n}$ is the direction of the outgoing wave,

d) For $ka \gg 1$, choose the statement that is closest to true:
(i) The differential cross section is highly peaked in the forward direction, and in particular quite large at $\theta = 0$.
(ii) The differential cross section is highly peaked in the forward direction, although zero at exactly $\theta = 0$.
(iii) The differential cross section still has broad support over a wide range of angles.
(iv) None of the above is close to true.

Answers:

a)

$$\frac{d\sigma}{d\Omega} =$$

b) Will find on work sheets.

c) $q =$ ______________________________

d) Write a roman numeral here: (   ).

2. [30 points total] Charges $+q$ and $-q$ exist at the ends of a neutral rod of length d. The rod rotates in the x-y plane with constant angular velocity ω about the z axis which passes through its center.

a) If we express the dipole moment as $\vec{p}(t) = \mathrm{Re}\,(\vec{p}e^{-i\omega t})$, find $\vec{p}$.

b) Compute the radiated power per unit solid angle $dP/d\Omega$ to leading non-trivial order (electric dipole radiation).

c) Find the total radiated power P.

Answers:

a) $\vec{p} =$

b)

$$\frac{dP}{d\Omega} =$$

c) $P =$

3. [40 points total]
(a) A rectangular waveguide made from a nearly perfect conductor has sides of length a and b, in the x and y directions, respectively. Consider the TM modes ($E_z \neq 0$) and starting with an expression like

$$E_z(x,y) = (A\cos k_x x + B\sin k_x x)\left(C\cos k_y y + D\sin k_y y\right)$$

find an expression for $E_z(x,y)$ that satisfies the boundary conditions. (If you get help from a footnote in Jackson, you should work out why the information found there is correct.)

(b) Give an expression for $\gamma^2 = \epsilon_0\mu_0\omega^2 - k^2$ (where k is the wavenumber in the z-direction in the full expression $E_z(x,y,z,t) = \mathrm{Re}\,E_z(x,y)e^{i(kz-\omega t)}$). Also specifically give γ^2_{lowest} for the lowest nontrivial mode.

(c) Find the transverse electric field $\vec{E}_t$.

(d) Discuss (briefly) how you would proceed to calculate the energy loss in the case that the conducting walls of the waveguide were not perfect conductors.

Answers:

a) $E_z(x,y) =$ ____ ,

b) $\gamma^2 =$ ____ ,

$\gamma^2_{\text{lowest}} =$ ____ ,

c) $\vec{E}_t =$ ____ .

d) Discussion: will find on work sheets.