Quantum Field Theory II **Physics 722**

Final Exam | Take home: three hours | officially 03 May 2017
due at latest 09 May 2017

Do not turn this page until you are ready to start the exam.

Rules:
1. You have three consecutive hours to work on the exam after you first look at the exam itself.
2. You can consult any of the following items: The text by Peskin and Schroeder, lecture notes you took during class, and your solutions to old homework problems from this course. You may open a computer for purposes of doing integrals if needed and/or latexing solutions if you are of such a mind. You may not consult other sources or other persons.
3. Please turn in the exams not later than 11:59 pm Tuesday, May 09, 2017. You may turn the exams in by giving them to me, by pushing them under my office door (Room 326C), or by email if scanned or latexed.

Carl Carlson

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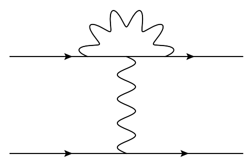
There are 5 problems, on two pages

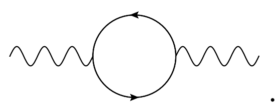
1. [20 points total] The Lagrangian of some non-Abelian gauge theory starts out as

$$\mathscr{L} = -\frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \sum_{i=1}^{2} \bar{\psi}_i \left(i\gamma^\mu \partial_\mu + g\gamma^\mu A^a_\mu t_a - m_i\right)\psi_i$$

for two different fermions ψ_1 and ψ_2. Draw the Feynman diagrams for all the one-loop corrections to $\psi_1\psi_2$ elastic scattering. Two diagrams are drawn to get you started, with ψ_1 the upper line and ψ_2 the lower line.



2. [20 points total] For the same non-Abelian gauge theory, consider the fermion loop corrections to the gauge boson propagator,



The corresponding correction in QED is

$$ie^2\,\Pi_{\mu\nu}(q) = ie^2\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right)\Pi(q^2,m),$$

where $\Pi(q^2,m)$ is a function you may assume you know.
What is the result for the analog of $\Pi_{\mu\nu}(q)$ in the non-Abelian gauge theory?
If you believe the answer is just a factor times the QED result, give the factor explicitly for the case that the group is $SU(3)$.
For simplicity, you may take the two fermions to have the same mass.

3. [20 points total] Consider the Lagrangian

$$\mathscr{L} = -\frac{1}{4}F^2_{\mu\nu} + \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi - \lambda\,\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu},$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where λ is a constant, and $\sigma_{\mu\nu}$ is a constant matrix antisymmetric in μ and ν.
Give the superficial degree of divergence for Feynman diagrams in this theory in d dimensions in terms of the numbers of external legs and, if necessary, the number of vertices in the diagram. Carefully consider that momentum factors associated with external legs do not contribute to the divergence of the diagrams.

4. [20 points total] Define the function $G(\theta-\eta) = \theta-\eta$, where both θ and η are Grassmann variables. Let $f(\theta)$ be a function of a Grassmann variable that can be Taylor expanded with c-number coefficients. Calculate the integral,

$$I = \int d\theta\; G(\theta-\eta)f(\theta).$$

Express the answer, as best you can, using the original function f.

5. [20 points total] A Lagrangian that will have spontaneous symmetry breaking is

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{1}{2}\mu^2 \vec{\phi}^2 - \frac{1}{4}\lambda \left(\vec{\phi}^2\right)^2,$$

with $\vec{\phi} = (\phi_1, \phi_2)$ and where the ϕ_i are real.

a) How can you tell, at a glance, that there will be spontaneous symmetry breaking?

b) Define two real functions ρ and θ by

$$\phi = (\phi_1 + i\phi_2)/\sqrt{2} = \rho e^{i\theta}.$$

Rewrite the above Lagrangian in terms of ρ and θ and derivatives upon them.

c) Shift ϕ according to

$$\rho = \sqrt{\frac{\mu^2}{2\lambda}} + \xi.$$

Verify that the non-derivative parts of the Lagrangian have an extremum at $\xi = 0$.

d) Write the Lagrangian in terms of θ and ξ (and constants). The results should look like

$$\mathscr{L} = const. + (\partial_\mu \xi)^2 + const. \times (\partial_\mu \theta)^2 + const. \times \xi^2 + \mathscr{L}_{\text{int}}.$$

Explicitly give $\mathscr{L}_{\text{int}}$.