

Physics 621 Homework Assignment 6

due Tuesday November 10 2009

Unless otherwise indicated, the problems are from the textbook (Merzbacher, 3rd edition).

1. Exercise 6.8 (pg 103). Aside: the optical theorem that is mentioned here is a very powerful result in scattering theory.
2. Problem 6.2 (pg 111). Aside: the δ here is the “scattering phase shift”, which encodes all the information on the scattering for a given incident energy.
3. In class we showed that the use of energy eigenstates Ψ_k leads to $\delta \langle H \rangle = 0$; prove that the converse also holds, *i.e.* that the states Ψ minimizing the variation must be energy eigenstates.
4. Let’s use the variational method to derive a very general result: we want to show that an arbitrarily weak attractive one-dimensional potential always has at least one bound state. Assume the potential is only non-zero in a limited region; take this region to be $|x| < a$ for $a =$ positive real constant. Choose a simple trial wavefunction, one that is continuous, constant inside the potential, and decays exponentially outside the potential. Use this to prove that there must be at least one bound state.
5. Consider a particle of mass m in a one-dimensional potential $V(x) = \lambda x^4$, where λ is a constant. Using the variational method, find an approximate value for the ground state energy; use a trial wave function $\psi = e^{-\alpha x^2}$. Compare your result to the exact value, which is $E_0 = 1.06 \frac{\hbar^4/3}{(2m)^{2/3}} \lambda^{1/3}$.