Physics 101 Test 1 Sept 26 2016

Name: SOLUTIONS

This test is administered under the rules and regulations of the Honor Code of William & Mary.

Signature: ____________________

Problem Session (circle one):

Wed. 9:00 am (Prof. Hoatson)

Wed. 3:00 pm (Prof. Vahala)

Wed. 3:00 pm (Prof. Hancock)

Thurs. 1:00 pm (Prof. Cooke)

Thurs. 3:30 pm (Prof Erlich)

Thurs. 2:00 pm (Prof. Carlson)

1. ________ (25 points)

2. ________ (25 points)

3. ________ (25 points)

4. ________ (25 points)

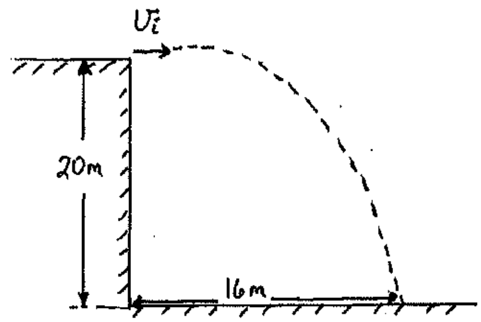
Total ________

Problem 1.

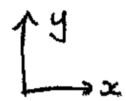
A ball is thrown horizontally from the roof of Swem library. The library is 20 m tall. The ball hits the ground a distance of 16 m from the base of the library. Neglect air resistance.

a) Find the initial speed with which the ball was thrown.

b) Find the magnitude and direction of the ball's velocity just before it hits the ground.



a) Projectile motion. Use $g \simeq 10\,m/s^2$ here.



with origin @ base of building

$$y_f = y_i + v_{iy}t - \frac{1}{2}gt^2 \qquad y_f = 0 \quad y_i = 20m \quad v_{iy} = 0$$

$$\therefore 0 = 20m + 0 - \frac{1}{2}gt^2$$

$$t^2 = \frac{20m \times 2}{g} = \frac{40m}{10m/s^2} = 4s^2 \qquad \therefore t = 2.0s$$

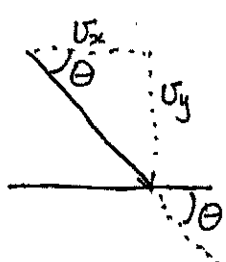
$$x_f = x_i + v_{ix}t \qquad x_i = 0 \quad x_f = 16m$$

$$v_{ix} = \frac{x_f - x_i}{t} = \frac{16m - 0m}{2.0s} = \boxed{8.0\,m/s} = v_i$$

b) $v_x = v_{ix} = 8.0\,m/s$

$$v_y = v_{iy} - gt = 0 - \left(10\frac{m}{s^2}\right)(2.0\,s) = -20\,m/s$$

$$v = |\vec{v}| = (v_x^2 + v_y^2)^{1/2} = (8^2 + (-20)^2)^{1/2} = \boxed{21.5\,m/s}$$



$$\tan\theta = \frac{|v_y|}{|v_x|} \qquad \theta = \tan^{-1}\left(\frac{20}{8}\right) = \boxed{68.2°}$$

Problem 2.

The position of an object of mass 0.2 kg varies in time according to $\vec{r} = (10.0t)\hat{\imath} - (7.5t^2)\hat{\jmath}$ where $\vec{r}$ is in meters and t is in seconds.

a) Find an expression for the velocity of the object as a function of time.
b) Determine the acceleration of the object as a function of time.
c) Calculate the object's position at $t = 2.0$ s.
d) Calculate the magnitude and direction of the object's velocity at $t = 2.0$ s. Make sure you clearly specify what angle you are using to define the direction.
e) What is the net force on the object at time $t = 2.0$ s?

a) $\vec{r} = 10t\,\hat{\imath} - 7.5t^2\,\hat{\jmath}$ $\quad \vec{v}(t) = \frac{d\vec{r}}{dt} = \boxed{10\hat{\imath} - 15t\,\hat{\jmath}}$ (in m/s)

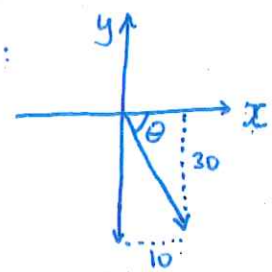
b) $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = 0\hat{\imath} - 15\hat{\jmath} = \boxed{-15\hat{\jmath}}$ (in m/s²)

c) $\vec{r}(t = 2.0\text{s}) = 10(2.0)\hat{\imath} - 7.5(2.0)^2\hat{\jmath} = \boxed{20\hat{\imath} - 30\hat{\jmath}}$ (in m)

d) $\vec{v}(t = 2.0\text{s}) = 10\hat{\imath} - 15(2.0)\hat{\jmath} = 10\hat{\imath} - 30\hat{\jmath}$

magnitude : $|\vec{v}| = (v_x^2 + v_y^2)^{1/2} = (10^2 + (-30)^2)^{1/2} = \boxed{31.6 \text{ m/s}}$

direction :



$\tan\theta = \frac{|v_y|}{|v_x|}$ $\quad \theta = \tan^{-1}\left(\frac{30}{10}\right) = \boxed{71.6^\circ}$

$\boxed{\text{clockwise below } +x \text{ axis}}$

e) $\sum\vec{F} = m\vec{a}$

$= (0.2\text{Kg})(-15\hat{\jmath}\ \frac{\text{m}}{\text{s}^2})$

$= \boxed{-3\text{ N}\hat{\jmath}}$

Problem 3.

A certain airplane can fly with a speed of 200 m/s relative to the air. If the air is moving at 30 m/s relative to the ground in the Northeast direction (45° North of East), then:

a) In what direction should the airplane fly, relative to the air, in order for the airplane to move due East relative to the ground?

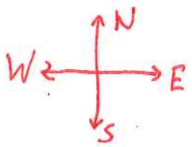
b) What is the magnitude of the airplane's speed while flying East relative to the ground?

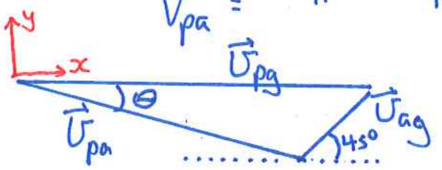
c) How much time is required for the airplane to fly a distance of 1000 km due East?

a) Relative Velocity

$\vec{V}_{ag}$ = velocity of air relative to ground
$\vec{V}_{pg}$ = " plane " " "
$\vec{V}_{pa}$ = " " " " air

$$\vec{V}_{pg} = \vec{U}_{pa} + \vec{U}_{ag}$$





$\vec{U}_{pg} = U_{pg}\hat{i}$ $\quad$ $\vec{U}_{ag} = U_{ag}\cos 45^\circ \hat{i} + U_{ag}\sin 45^\circ \hat{j}$ $\quad$ $\vec{U}_{pa} = U_{pa}\cos\theta\,\hat{i} - U_{pa}\sin\theta\,\hat{j}$

x-component : $U_{pg} = U_{pa}\cos\theta + U_{ag}\cos 45^\circ$

y-component : $0 = -U_{pa}\sin\theta + U_{ag}\sin 45^\circ \longrightarrow \sin\theta = \frac{U_{ag}}{U_{pa}}\sin 45^\circ = \frac{30\,m/s}{200\,m/s}\frac{1}{\sqrt{2}}$

$$= 0.106$$

$$\theta = \sin^{-1}(0.106) = \boxed{6.09^\circ}$$

SOUTH OF EAST
(see sketch)

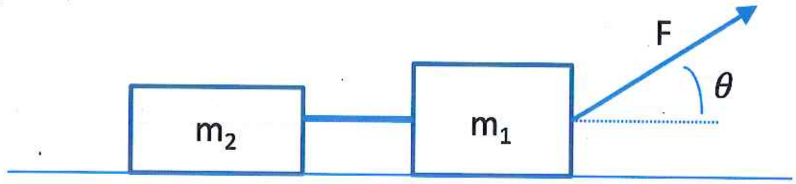
b) $U_{pg} = U_{pa}\cos(6.09^\circ) + U_{ag}\cos 45^\circ$

$$= \left(200\frac{m}{s}\right)(0.994) + \left(30\frac{m}{s}\right)\frac{1}{\sqrt{2}} = \boxed{220\ m/s}$$

c) $U_{pg} = \dfrac{\Delta x}{\Delta t}$ $\quad \therefore \quad \Delta t = \dfrac{\Delta x}{U_{pg}} = \dfrac{1000\times10^3\,m}{220\,m/s} = \boxed{4545\,s}$

$$= 1.26\ hr$$

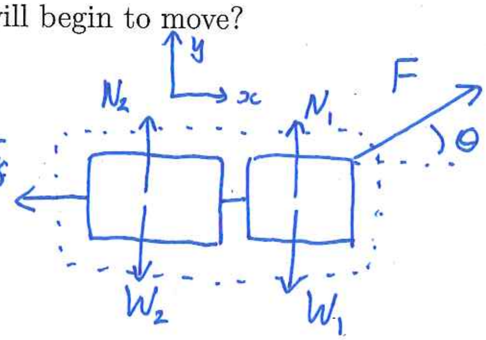
Problem 4.



Two masses, $m_1 = 3$ kg and $m_2 = 2$ kg are on a horizontal surface and are connected together via a massless rope. An external force F pulls on m_1 at an angle of $\theta = 30°$ from the horizontal, as shown. There is friction between mass m_2 and the surface, with coefficients $\mu_s = 0.4$ and $\mu_k = 0.3$, however the friction between m_1 and the surface is negligibly small.

a) What is the minimum value of F such that the masses will begin to move?

b) If, instead, $F = 30$ N, what is the tension in the rope?



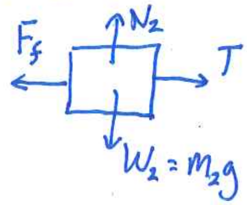
a) Consider m_1 & m_2 as one system. FBD :

constraints : $W_2 = m_2 g$ $W_1 = m_1 g$ $F_f \leq \mu_s N_2$

minimum F when $a \geq 0$ F_f is maximum $\therefore F_f = \mu_s N_2$

$$\therefore \sum_i \vec{F}_i = m\vec{a} = 0 \quad \begin{cases} x: & F^{min}\cos\theta - F_f = 0 \\ y: & F\sin\theta + N_2 + N_1 - W_2 - W_1 = 0 \end{cases} \longrightarrow F^{min}\cos\theta = F_f = \mu_s N_2$$

to get N_2 consider FBD on m_2 alone :



y-component gives $N_2 = m_2 g$

$$\therefore F^{min}\cos\theta = \mu_s m_2 g$$

$$\therefore F^{min} = \frac{\mu_s m_2 g}{\cos\theta} = \frac{(0.4)(2\,\text{Kg})(9.8\,\text{m/s}^2)}{\cos 30°} = \boxed{9.0\text{ N}}$$

b) Now, $\vec{a} \neq 0$

Now use μ_K, not μ_s (in motion)

FBD on 2-mass system gives (x-component):

$$F\cos\theta - \mu_K m_2 g = (m_1 + m_2)a$$

$$a = \frac{F\cos\theta - \mu_K m_2 g}{(m_1 + m_2)} = \frac{(30\text{N})(\cos 30°) - (0.3)(2\,\text{Kg})(9.8\,\text{m/s}^2)}{(3\,\text{Kg} + 2\,\text{Kg})}$$

$$= 4.0\,\text{m/s}^2$$

FBD on m_2 gives (x-component)

$$T - F_f = m_2 a \quad \therefore \quad T = m_2 a + \mu_K m_2 g$$

$$= m_2(a + \mu_K g) = 2\,\text{Kg}(4.0\,\text{m/s}^2 + 0.3(9.8\,\text{m/s}^2)) = \boxed{13.9\text{ N}}$$