

Physics 108

Test 2

March 18 2016

Name: SOLUTIONS

This test is administered under the rules and regulations
of the Honor Code of William & Mary.

Signature: _____

1. _____ (25 points)

2. _____ (25 points)

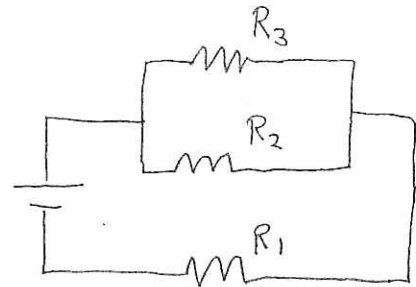
3. _____ (20 points)

4. _____ (30 points)

Total _____

Problem 1. [25 points]

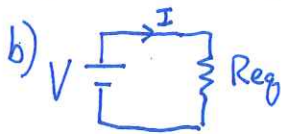
Consider the circuit shown, where $V = 10\text{ V}$, $R_1 = 3\text{ k}\Omega$, $R_2 = 10\text{ k}\Omega$, and $R_3 = 10\text{ k}\Omega$.



- [7 points] What is the total equivalent resistance of the three resistors?
- [8 points] What is the current through each resistor?
- [5 points] How much energy is dissipated as heat in resistor R_1 in 10 seconds?
- [5 points] Assume that the resistors are actually the filaments of incandescent light bulbs. If R_2 is disconnected from the circuit, will the R_1 bulb get brighter or dimmer? Explain your reasoning.

$$a) \frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{10\text{k}\Omega} + \frac{1}{10\text{k}\Omega} = \frac{1}{5\text{k}\Omega} \quad \therefore R_p = 5\text{ k}\Omega$$

this is in series with $R_1 \quad \therefore R_{eq} = R_1 + R_p = 3\text{ k}\Omega + 5\text{ k}\Omega = \boxed{8\text{ k}\Omega}$



$$I = \frac{V}{R_{eq}} = \frac{10\text{V}}{8\text{k}\Omega} = 1.25 \times 10^{-3}\text{ A}$$

$$\boxed{I_1 = I = 1.25 \times 10^{-3}\text{ A}}$$

$$V_1 = I_1 R_1 = (1.25 \times 10^{-3}\text{ A})(3 \times 10^3 \Omega) = 3.75\text{ V}$$

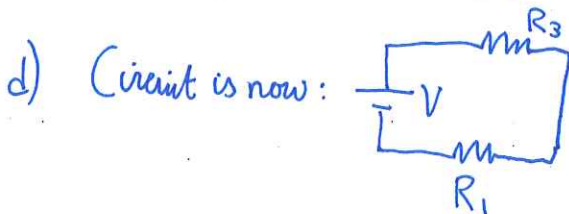
$$\therefore V_2 = V_3 = (V - V_1) = 10\text{V} - 3.75\text{V} = 6.25\text{V}$$

$$I_2 = \frac{V_2}{R_2} = \frac{6.25\text{V}}{10^4 \Omega} = \boxed{6.25 \times 10^{-4}\text{ A}}$$

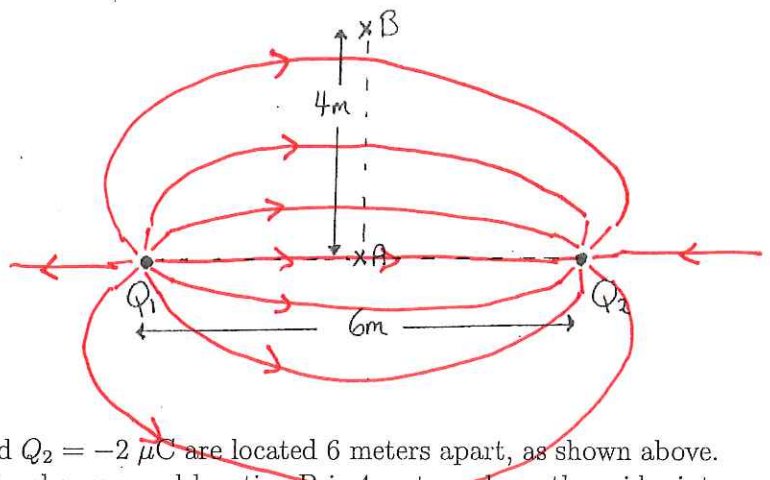
$$I_3 = \frac{V_3}{R_3} = \text{''} = \text{''}$$

(or : since $R_2 = R_3$ must have $I_2 = I_3$
& $I_2 + I_3 = I_1 \quad \therefore I_2 = I_3 = I_1/2 \dots$)

$$c) P_1 = I_1^2 R_1 = (1.25 \times 10^{-3}\text{ A})^2 (3 \times 10^3 \Omega) = 4.69 \times 10^{-3}\text{ W} \quad E = Pt = (4.69 \times 10^{-3}\text{ W})(10\text{s}) = \boxed{4.69 \times 10^{-2}\text{ J}}$$



Since R_2 is removed, the resistance of what was the parallel combination has increased (is now $10\text{ k}\Omega$, it was $5\text{ k}\Omega$) \therefore total R_{eq} has increased (it is now $R_2 + R_3 = 13\text{ k}\Omega$ instead of $8\text{ k}\Omega$)
Thus the current $I = I_1$ has decreased (V is the same) \rightarrow therefore the power has decreased, thus the bulb is dimmer



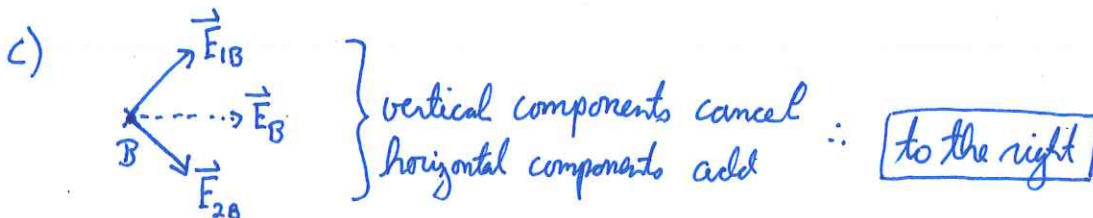
Problem 2. [25 points]

Two charges, $Q_1 = +2 \mu\text{C}$ and $Q_2 = -2 \mu\text{C}$ are located 6 meters apart, as shown above. Location A is midway between the charges, and location B is 4 meters above the midpoint.

- [6 points] Find the electric field (magnitude and direction) at location A.
- [5 points] What would be the force (magnitude & direction) on an electron if it was located at A?
- [4 points] What is the direction (don't worry about the magnitude) of the electric field at location B?
- [6 points] What is the electric potential at location B?
- [4 points] Sketch at least 8 electric field lines generated by these charges.

$$\begin{aligned}
 \text{a) } E_1 &= \frac{kQ_1}{r_1^2} = \frac{(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(2 \times 10^{-6} \text{C})}{(3\text{m})^2} = 2000 \text{ N/C}; \text{ to the right} \\
 E_2 &= \frac{kQ_2}{r_2^2} = \frac{(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(2 \times 10^{-6} \text{C})}{(3\text{m})^2} = \text{ " " }
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Total: } \vec{E}_A = \vec{E}_1 + \vec{E}_2 = \boxed{4000 \text{ N/C, to the right}}$$

$$\text{b) } |F| = |q_e| E_A = (1.6 \times 10^{-19} \text{C})(4000 \frac{\text{N}}{\text{C}}) = \boxed{6.4 \times 10^{-16} \text{N}} \quad \boxed{\text{to the left}} \quad (\vec{F} = q\vec{E} \text{ \& } q = \text{negative})$$



$$\text{d) } V_B = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} \quad r_1 = r_2 = 5\text{m here} \quad \therefore V_B = \frac{k}{(5\text{m})^2} [Q_1 + Q_2] = \frac{k}{(5\text{m})^2} [2\mu\text{C} - 2\mu\text{C}] = \boxed{0\text{V}}$$

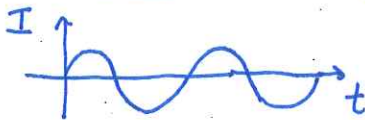
- e) see sketch above;
 - lines run from + to - charges; equal number leave as arrive (equal charges); lines don't cross...

3. [20 points] (these are three unrelated problems)

a) [7 points] You are toasting your breakfast bagel using a 3.2 amp DC current entering your toaster. The current is on for 10 seconds. How many electrons go through the toaster during this time? If it was, instead, a 3.2 amp AC current, how many electrons go through the toaster?

$$\text{DC: } I = \frac{\Delta Q}{\Delta t} \quad \therefore \Delta Q = I \Delta t = (3.2 \text{ A})(10 \text{ s}) = 32 \text{ C}$$
$$Q = n|q_e| \quad \text{where } n = \# \text{ electrons} \quad \therefore n = \frac{Q}{|q_e|} = \frac{32 \text{ C}}{1.6 \times 10^{-19} \text{ C}} = \boxed{2 \times 10^{20}}$$

AC: \approx Zero electrons \rightarrow charge sloshes back and forth, but no net charge moves through toaster



b) [7 points] A $3 \mu\text{F}$ capacitor is charged to a voltage of 20 V. If it is then discharged through a $100 \text{ M}\Omega$ resistor, how long does it take for the voltage on the capacitor to fall to 1 V?

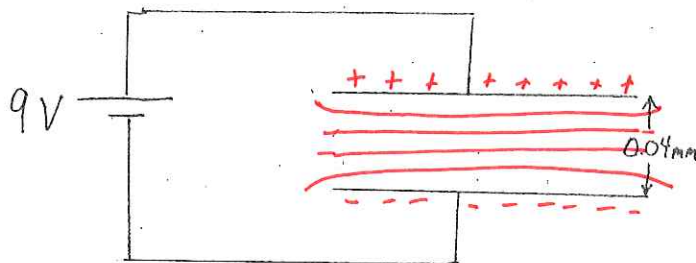
$$\tau = RC = (100 \times 10^6 \Omega)(3 \times 10^{-6} \text{ F}) = 300 \text{ s}$$
$$V(t) = V_0 e^{-t/\tau}$$
$$1 \text{ V} = (20 \text{ V}) e^{-t/\tau} \quad \therefore e^{-t/\tau} = \frac{1}{20} \quad \ln\left(\frac{1}{20}\right) = -t/\tau$$
$$\therefore t = -\tau \ln\left(\frac{1}{20}\right)$$
$$= -(300 \text{ s}) \ln(0.05)$$
$$= \boxed{899 \text{ s}} \quad = 15 \text{ min.}$$

c) [6 points] You have connected a Galvanometer in series with a $10 \text{ M}\Omega$ resistor. Is the resulting device useful as an Ammeter or as a Voltmeter?

Voltmeters are made using galvanometers in series with a large resistance.

Problem 4. [30 points]

The plates of a parallel-plate capacitor each have an area of 16 cm^2 , and they are separated by 0.04 mm . It is connected to a 9 V battery as shown.



- [5 points] What is the magnitude of the charge on each plate?
- [5 points] On the diagram, draw several equipotential lines in the region between the capacitor plates.
- [5 points] Which way (up, down, left, or right) does the electric field between the plates point?
- [5 points] What is the magnitude of the electric field between the plates?
- [5 points] The battery is disconnected from the circuit (without allowing the capacitor to discharge). Now, a sheet of paraffin (dielectric constant = 2.2) is inserted between the plates; it fills the space completely. What is the new charge and the new voltage on the capacitor?
- [5 points] Prof. Armstrong takes a much larger capacitor, $C = 100 \mu\text{F}$, and charges it using a 250 V power supply. He then disconnects it from the power supply, and connects a metal screwdriver across the plates. All the stored energy is discharged in 10^{-6} s with a loud bang. What is the power (in watts) of this discharge?

a) $C = k \frac{\epsilon_0 A}{d}$ $k=1$ for air

$$C = (1) (8.85 \times 10^{-12} \text{ F/m}) \left(\frac{16 \times 10^{-4} \text{ m}^2}{0.04 \times 10^{-3} \text{ m}} \right) = 3.54 \times 10^{-10} \text{ F} = 354 \text{ pF}$$

$$Q = CV = (3.54 \times 10^{-10} \text{ F})(9 \text{ V}) = \boxed{3.19 \times 10^{-9} \text{ C}}$$

b) see diagram; equally spaced parallel lines

c) \vec{E} points down (away from positive charges)

d) Uniform field: $V = Ed \therefore E = \frac{V}{d} = \frac{9 \text{ V}}{0.04 \times 10^{-3} \text{ m}} = \boxed{2.25 \times 10^5 \text{ V/m}} = 225 \text{ KV/m}$

e) $C_{\text{new}} = k \frac{\epsilon_0 A}{d} = kC = (2.2)(3.54 \times 10^{-10} \text{ F}) = 7.79 \times 10^{-10} \text{ F}$

$$Q_{\text{new}} = C_{\text{new}} V_{\text{new}} \therefore V_{\text{new}} = \frac{Q_{\text{new}}}{C_{\text{new}}} = \frac{3.19 \times 10^{-9} \text{ C}}{7.79 \times 10^{-10} \text{ F}} = \boxed{4.09 \text{ V}}$$

$Q_{\text{new}} = Q$ (no discharge!)
(disconnected from battery - nowhere for charge to go) = $\boxed{3.19 \times 10^{-9} \text{ C}}$

f) $E_{\text{cap}} = \frac{1}{2} CV^2 = \frac{1}{2} (100 \times 10^{-6} \text{ F})(250 \text{ V})^2 = 3.12 \text{ J}$

$$\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{3.12 \text{ J}}{10^{-6} \text{ s}} = \boxed{3.12 \times 10^6 \text{ W}}$$

3 Megawatts!