Problem 1. [25 points]

A single ice cube has a mass of 20 grams. It is initally immersed in liquid nitrogen in order to bring its temperature to -195°C. It is then placed in one half liter of water (mass of 500 grams) which is initially at room temperature, 20°C, and is in an insulated container. What is the final temperature of the system? *Hint:* all the ice will melt.

Possibly useful constants: $c_{\text{ice}} = 0.50$ kcal/kg·°C, $c_{\text{water}} = 1.00$ kcal/kg·°C, $L_f^{\text{water}} = 80$ kcal/kg, $L_v^{\text{water}} = 539$ kcal/kg.

Calorimetry Problem : $Q_{hot} + Q_{cold} = 0$

$$Q_{hot} = Q_1 + Q_2 + Q_3 \quad \left\{ \begin{array}{l} Q_1 = \text{raise ice to melting point} = m_i c_i (0°C - (-195°C)) \\ Q_2 = \text{melt ice} = m_i L_f \\ Q_3 = m_i c_w (T - 0°C) \end{array} \right.$$

$$Q_{cold} = m_w c_w (T - 20°C)$$

$$Q_{hot} + Q_{cold} = 0 \qquad m_i c_i (195°C) + m_i L_f + m_i c_w T + m_w c_w (T - 20°C) = 0$$

$$T(m_i c_w + m_w c_w) = m_w c_w (20°C) - m_i c_i (195°C) - m_i L_f$$

$$T = \frac{m_w c_w (20°C) - m_i c_i (195°C) - m_i L_f}{m_i c_w + m_w c_w}$$

$$= \frac{(0.5\,\text{Kg})\left(1.0\,\frac{\text{Kcal}}{\text{Kg}\,°C}\right)(20°C) - (2\times10^{-2}\,\text{Kg})\left(0.5\,\frac{\text{Kcal}}{\text{Kg}\,°C}\right)(195°C) - (2\times10^{-2}\,\text{Kg})\left(80\,\frac{\text{Kcal}}{\text{Kg}}\right)}{(2\times10^{-2}\,\text{Kg})\left(1.0\,\frac{\text{Kcal}}{\text{Kg}\cdot°C}\right) + (0.5\,\text{Kg})\left(1.0\,\frac{\text{Kcal}}{\text{Kg}\cdot°C}\right)}$$

$$= \boxed{12.4°C}$$

(see lecture notes from Jan 27th & problem #3 on Homework 2)

Problem 2. [20 points]

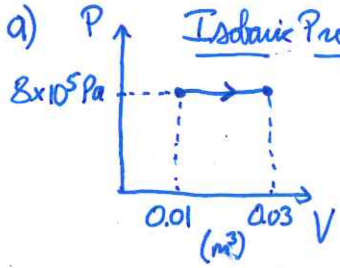
Two moles of an ideal diatomic gas expands from 0.01 m^3 to 0.03 m^3 at a constant pressure of 800 kilopascals.

a)[5 points] Sketch the process on a P-V diagram.

b)[7 points] How much work is done by the gas during this expansion?

c)[8 points] How much heat flows into the gas during this expansion?

a)



b) $W = P\Delta V = P(V_f - V_i) = (8\times10^5\,Pa)(0.03\,m^3 - 0.01\,m^3)$
$= \boxed{+1.6\times10^4\,J}$

c) 1st law : $\Delta U = Q - W \quad \therefore Q = \Delta U + W$

$PV = nRT \rightsquigarrow P\Delta V = nR\Delta T$

$\Delta U = \frac{5}{2} nR\Delta T = \frac{5}{2} P\Delta V$

$Q = \Delta U + W = \frac{5}{2}P\Delta V + P\Delta V = \frac{7}{2}P\Delta V = \frac{7}{2}(1.6\times10^4\,J)$
$= \boxed{5.6\times10^4\,J}$

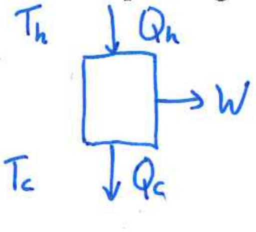
Problem 3. [15 points]

A heat engine operates using the Carnot cycle. Every second, it produces 200 J of work, while dumping 500 J of waste heat into a cold reservoir, which is at room temperature (20°C).

a)[10 points] What is the temperature of the hot reservoir?

b)[5 points] What would be the coefficient of performance of a Carnot-cycle refrigerator operating between these two temperatures?

$T_c = 20°C = 293\,K$



a) $Q_h = Q_c + W = 200J + 500J = 700J$

$\varepsilon = \frac{W}{Q_h} = 1 - T_c/T_h$

$= \frac{200J}{700J} = \text{"} \quad \therefore 1 - \frac{T_c}{T_h} = \frac{2}{7} \quad \therefore \frac{T_c}{T_h} = 1 - \frac{2}{7} = \frac{5}{7}$

$\therefore T_h = \frac{7}{5}T_c$
$= \frac{7}{5}(293\,K)$
$= \boxed{410\,K}$

b) $COP^{ref} = \frac{Q_c}{W} = \frac{500J}{200J} = \boxed{2.5}$

Problem 4. [15 points]

A 300.0 cm long piece of an unknown metal, which is at room temperature (20°C) is given to you. You suspect that it is either made of steel or aluminum. You heat it up to 100°C and find that the length of the piece is now 300.6 cm. The coefficient of linear expansion for steel is $25 \times 10^{-6} C^{-1}$ and the coeffcient for aluminum is $12 \times 10^{-6} C^{-1}$.

Is the piece made of steel or aluminum? Show your calculation.

$$\Delta L = \alpha L \Delta T$$

$$\alpha = \frac{\Delta L}{L \Delta T} = \frac{(300.6 - 300.0)\,cm}{(300\,cm)(100^\circ C - 20^\circ C)} = 2.5 \times 10^{-5}\,C^{-1} \quad \boxed{\therefore \text{steel}}$$

(see, e.g., problem #3 on Homework #1)

Problem 5. [15 points]

In one minute, how much heat is transferred by radiation out of a car radiator at 110°C into a 50°C environment? The radiator has an emissivity of 0.750 and a 1.20 m^2 surface area.

$$\frac{Q}{t} = \sigma \varepsilon A (T_{hot}^4 - T_{cold}^4)$$

$T_{hot} = 110^\circ C = 383K$

$T_{cold} = 50^\circ C = 323K$

$$\frac{Q}{t} = \left(5.69 \times 10^{-8} \frac{J}{s\,m^2 \cdot K^4}\right)(0.750)(1.2\,m^2)\left[(383\,K)^4 - (323\,K)^4\right]$$

$$= 544\,W$$

$$Q = \frac{Q}{t} \cdot t = (544\,W)(60\,s) = \left(544\,\frac{J}{s}\right)(60\,s) = \boxed{32.7\,kJ}$$

(see lecture notes; example done Jan. 29)

Problem 6. [10 points]

Suppose that the average velocity ($v_{\rm rms}$) of carbon dioxide molecules (molecular mass 44.0 grams per mole) in a flame is found to be 1.00×10^5 m/s.

a) [5 points] What temperature does this represent?

b) [5 points] If the average velocity were, instead, 1/4 of this value, what temperature would this now represent?

a) $U_{rms} = \left(\frac{3KT}{m}\right)^{1/2} \quad \therefore \quad U_{rms}^2 = \frac{3KT}{m}$

$$\therefore T = \frac{mU_{rms}^2}{3K} = \frac{(44\times10^{-3}\ \mathrm{Kg/mol})\left(\frac{1\ \mathrm{mol}}{6.0\times10^{23}}\right)(10^5\ \mathrm{m/s})^2}{3(1.38\times10^{-23}\ \mathrm{J/K})}$$

(see Problem #10 on Homework #1)

$$= \boxed{1.77\times10^{7}\ \mathrm{K}}$$

Wow! Hot stuff!

b) $T \propto (U_{rms})^2$

$\therefore \quad \left(\frac{1}{4}\right)^2 = \frac{1}{16} \quad \therefore \quad T_{new} = \frac{1}{16}T = \boxed{1.11\times10^{6}\ \mathrm{K}}$

still rather hot!