Problem 1.

Prof. Armstrong's green laser pointer emits light of wavelength 532 nanometers, and the power is 5 milliWatts.

a) The laser beam impinges on a spot 3 mm in diameter. What is the average intensity of the light in the spot?

b) What is the maximum electric field strength of the light?

c) What is the energy of a single photon of this light? Give the answer in both units of Joules and electron-volts.

d) What is the momentum of a single photon of this light?

e) How many photons are emitted by the laser in one minute?

a) $I = P/A \qquad A = \pi r^2 = \pi (d/2)^2 = \frac{\pi}{4}(3\times10^{-3}m)^2 = 7.07\times10^{-6}m^2$

$\therefore I = \frac{5\times10^{-3}W}{7.07\times10^{-6}m^2} = \boxed{707\ W/m^2}$

b) $I = \langle u \rangle c = \varepsilon_0 E_{rms}^2 c \qquad \therefore E_{rms} = \left(\frac{I}{\varepsilon_0 c}\right)^{1/2} = \left[\frac{707\ W/m^2}{(3\times10^8 \frac{m}{s})(8.85\times10^{-12} F/m)}\right]^{1/2}$

$= 516\ V/m$

but max value = peak = $\sqrt{2}\, E_{rms} \quad \therefore \quad E_0 = \sqrt{2}\ 516\ V/m = \boxed{730\ V/m}$

c) $E = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34} J.s)(3\times10^8 m/s)}{532\times10^{-9}m} = \boxed{3.74\times10^{-19} J}$

$3.74\times10^{-19} J \times \frac{1 eV}{1.6\times10^{-19} J} = \boxed{2.34\ eV}$

d) $p = \frac{E}{c} = \frac{3.74\times10^{-19} J}{3\times10^8 m/s} = \boxed{1.25\times10^{-27} Kg\frac{m}{s}}$

e) $P = \frac{E}{t} \quad \therefore \quad P = \frac{E}{\gamma}\cdot\frac{\gamma}{t}$

$\therefore \frac{\gamma}{t} = \frac{P}{E/\gamma} = \frac{5\times10^{-3}W}{3.74\times10^{-19}J} = 1.34\times10^{16}\ \gamma/s$

$1.34\times10^{16}\frac{\gamma}{s}\times\frac{60 s}{min} = \boxed{8.02\times10^{17}\ \gamma s}$ in 1 minute

Problem 2.

The component of the external magnetic field along the central axis of a 50-turn coil of radius 5.5 cm increases from 0 T to 1.8 T in 4.0 s.

a) If the resistance of the coil is 2.8 Ω , what is the magnitude of the induced current in the coil?

b) What is the direction of the current if the axial component of the field points towards the viewer? Your choices are: clockwise or counterclockwise.

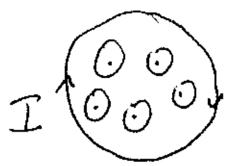
a) $$|\mathcal{E}| = N\frac{\Delta\phi}{\Delta t} = NA\frac{\Delta B}{\Delta t} = 50\,\pi(5.5\times10^{-2}\,m)^2\frac{(1.8T - 0T)}{4.0s}$$

$A = \pi r^2$

$$= 0.214\ V$$

$$|\mathcal{E}| = Ir \quad \therefore \quad I = \frac{|\mathcal{E}|}{r} = \frac{0.214V}{2.8\Omega} = \boxed{0.076\ A} = 76\ mA$$

b)



ϕ comes out of page & is increasing
$\therefore$ Lenz's law says $\mathcal{E}$ is such as to induce a current to oppose change $\therefore$ put flux into page
$\therefore$ Clockwise current

Problem 3.

23 **V** Vanadium 50.94	24 **Cr** Chromium 52.00	25 **Mn** Manganese 54.94	26 **Fe** Iron 55.85	27 **Co** Cobalt 58.93
41 **Nb** Niobium 92.91	42 **Mo** Molybdenum 95.94	43 **Tc** Technetium (98)	44 **Ru** Ruthenium 101.1	45 **Rh** Rhodium 102.9
73 **Ta** Tantalum 180.9	74 **W** Tungsten 183.8	75 **Re** Rhenium 186.2	76 **Os** Osmium 190.2	77 **Ir** Iridium 192.2
105 **Db** Dubnium (262)	106 **Sg** Seaborgium (263)	107 **Bh** Bohrium (262)	108 **Hs** Hassium (265)	109 **Mt** Meitnerium (266)

^{99}Tc is a β^- emitting nuclide that is widely used in medical diagnostic tests. It has a half-life of about 6 hours.

a) What is the daughter nuclide (answer in the form $^A_Z X_N$)? A relevant portion of the periodic table is shown.

b) You have a sample with an activity of 16 microcuries (recall: a Curie is 3.7×10^{10} disintegrations per second). How many ^{99}Tc atoms are in the sample?

c) The sample must have an activity of at least 2 microcuries to be effective in your diagnostic scan. How long do you have before it becomes ineffective?

d) If one of the electrons that is emitted is travelling at 84% of the speed of light, what is its wavelength? The mass of an electron is 9.11×10^{-31} kg, or 511 keV/c^2.

a) $$^{99}_{43}Tc_{56} \longrightarrow \boxed{^{99}_{44}Ru_{55}} + \beta^- + \overline{\nu_e}$$

b) $$R = \frac{0.693 N}{t_{1/2}} \qquad t_{1/2} = 6\,\text{hours} \times \frac{3600\,s}{\text{hour}} = 2.16 \times 10^4\,s$$

$$R = (16 \times 10^{-6}\,Ci)\left(3.7 \times 10^{10}\,\frac{s^{-1}}{Ci}\right) = 5.92 \times 10^5\,s^{-1} = 5.92 \times 10^5\,Bq$$

$$N = \frac{R\,t_{1/2}}{0.693} = \frac{(5.92 \times 10^5\,s^{-1})(2.16 \times 10^4\,s)}{0.693} = \boxed{1.84 \times 10^{10}}$$

c) $$2 = 16 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \quad \therefore 3 \text{ half lifes} \quad \therefore 3 \times 6\,\text{hours} = \boxed{18\,\text{hours}}$$

or $$R = R_0 e^{-\lambda t} \qquad \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.16 \times 10^4 s} = 3.21 \times 10^{-5}\,s^{-1}$$

$$\frac{R}{R_0} = e^{-\lambda t} \qquad \ln(R/R_0) = -\lambda t \quad \therefore \quad t = -\frac{1}{\lambda}\ln(R/R_0) = \frac{-1}{3.21 \times 10^{-5}\,s^{-1}} \ln\left(\frac{2\,\mu C}{16\,\mu C}\right)$$

$$= 6.48 \times 10^4\,s = 18\,\text{hours}$$

d) $$p = mv^{*} \quad \lambda = h/p = \frac{h}{mv}$$

$$= \frac{6.626 \times 10^{-34}\,J \cdot s}{(9.11 \times 10^{-31}\,kg)(3 \times 10^8\,m/s)(0.84)} = \boxed{2.88 \times 10^{-12}\,m}$$

STRICTLY SPEAKING, THIS EQUATION
EAKS DOWN WHEN v IS NEAR
, AS IT IS HERE... BUT THE CORRECT
UATION IS BEYOND THE SCOPE OF
HYSICS 108...

Problem 4

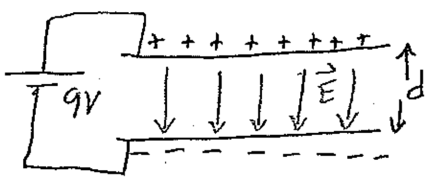
Two parallel metal plates are connected to a battery which supplies a 9.0 V potential difference between the two plates. The plates are oriented horizontally, with the upper plate having a charge of $+1.8 \times 10^{-11}$ C and the lower plate having a charge of -1.8×10^{-11} C. Each plate has an area of 2.0 cm^2. The space between them is evacuated.

a) What is the capacitance of this capacitor?

b) What is the distance between the plates?

c) What is the electric field (magnitude and direction) in the region between the plates?

d) A speck of dust is observed to float suspended somewhere between the plates; the speck has a charge of -1.0×10^{-14} C. What is the mass of the speck?

a) 

$Q = CV \quad \therefore \quad C = Q/V$

$= 1.8\times10^{-11}\,C / 9.0\,V$

$= \boxed{2\times10^{-12}\,F} = 2\,pF$

b) $C = K\varepsilon_0 \frac{A}{d}$ (K=1 here)

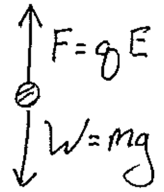
$\therefore \quad d = \frac{K\varepsilon_0 A}{C} = \frac{\varepsilon_0 A}{C} = \frac{(8.85\times10^{-12}\,F/m)\left(2.0\,cm^2 \times \left(\frac{1m}{10^2 cm}\right)^2\right)}{2\times10^{-12}\,F}$

$= \boxed{8.85\times10^{-4}\,m} = 0.885\,mm$

c) E = constant here $= \frac{\Delta V}{\Delta x}$

$\therefore \Delta V = E\Delta x = Ed \quad \therefore \quad E = V/d = \frac{9.0\,V}{8.85\times10^{-4}\,m} = \boxed{1.02\times10^{4}\,V/m}$

direction of $\vec{E}$ = $\boxed{\text{downward}}$

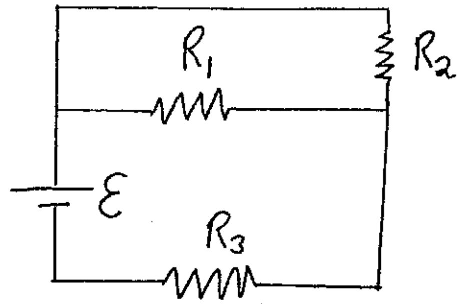
d) Free-Body diagram : 

$\vec{a} = 0 \quad \therefore \quad qE - mg = 0$

$\therefore \quad m = \frac{qE}{g} = \frac{(10^{-14}\,C)(1.02\times10^{4}\,N/C)}{9.8\,m/s^2}$

$= \boxed{1.04\times10^{-11}\,Kg} = 10.4\,pg$

Problem 5.



Consider the circuit shown. $R_1 = 300\ \Omega$, $R_2 = 600\ \Omega$, and $\mathcal{E} = 9$ V.

a) I_3 (the current through R_3) is measured to be 30 mA. What is the resistance of R_3?

b) How many electrons pass through R_3 in one hour?

c) If R_2 is removed from the circuit, would the current in R_3 increase or decrease?

d) R_3 is made from a cylindrical piece of material of length 10 cm and diameter 0.8 mm. What is the resistivity of this material?

a) R_3 is in series with the battery, $\mathcal{E}$, $\therefore I_3 = I_\mathcal{E}$

$$\mathcal{E} = I_\mathcal{E} R_{eq} \quad \therefore \quad R_{eq} = \frac{\mathcal{E}}{I_\mathcal{E}}$$

R_1 & R_2 in parallel ; R_3 in series with the R_1 & R_2 combination

$$\therefore \quad R_{eq} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} + R_3 = \frac{\mathcal{E}}{I_\mathcal{E}} \quad \therefore \quad R_3 = \frac{\mathcal{E}}{I_\mathcal{E}} - \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

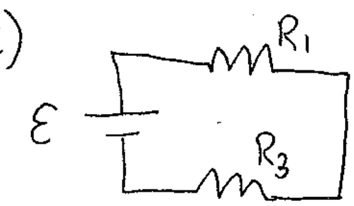
$$= \frac{9V}{30\times10^{-3}A} - \frac{1}{\left(\frac{1}{300\Omega} + \frac{1}{600\Omega}\right)}$$

$$= 300\Omega - 200\Omega = \boxed{100\Omega}$$

b) $$I = \frac{\Delta Q}{\Delta t} = \frac{e N_e}{\Delta t}$$

$$N_e = \frac{I \Delta t}{e} = \frac{(30\times10^{-3}A)(3600s)}{1.6\times10^{-19}C} = \boxed{6.75\times10^{20}}$$

c)



now, $R_{eq} = R_1 + R_3 = 400\Omega$ which is larger than original R_{eq} $\therefore$ current in I_3 (= current $I_\mathcal{E}$) would $\boxed{\text{decrease}}$

d) $$R_3 = \frac{\rho l}{A} \quad \therefore \quad \rho = \frac{R_3 A}{l} = \frac{(100\Omega)\pi\left(\frac{d}{2}\right)^2}{l}$$

$$= \frac{(100\Omega)\pi\left(\frac{0.8\times10^{-3}m}{2}\right)^2}{0.10m}$$

$$= \boxed{5.03\times10^{-4}\ \Omega m}$$

Problem 6.

You have a small refrigerator in your room. The refrigerator has a coefficient of performance of 3.0. Possibly useful data for water: specific heat = 4186 J/kg·°C, latent heat of fusion = 334 kJ/kg, latent heat of vaporization = 2256 kJ/kg, thermal conductivity = 0.6 J/s·m·°C.

a) How many kilowatt-hours of electrical power must be supplied to the refrigerator to remove 42 kJ of heat from the inside of the fridge?

b) How many grams of water, originally at 20°, could be frozen by removing this amount of heat?

c) The refrigerator moves heat from the cold inside to the hotter outside. Explain, in one or two sentences, why this process does *not* violate the second law of thermodynamics.

d) If the inside of the fridge is at -10° and the outside is at 20°, calculate the rate that heat escapes through one wall of the fridge, given that the wall is 0.5 cm thick, 20 cm wide, 30 cm tall, and is made from Aluminum, which has a thermal conductivity of 220 J/s·m·°C.

a) $C.O.P.^{ref} = \frac{Q_c}{W} \quad \therefore \; W = \frac{Q_c}{COP} = \frac{42\times10^3 J}{3} = 14\times10^3 J$

$1 \text{ Kilowatt·hour} = 10^3 W \cdot 3600 s = 3.6\times10^6 J$

$\therefore \; W = 14\times10^3 J \times \frac{1 \text{ KW hour}}{3.6\times10^6 J} = \boxed{3.89\times10^{-3} \text{ KW hour}}$

b) $Q_c = mL_f + mc\Delta T$

$\therefore \; m = \frac{Q_c}{(L_f + c\Delta T)} = \frac{42 \text{ KJ}}{\left(334 \frac{KJ}{Kg} + \left(4.186 \frac{KJ}{Kg}\right)(20°C)\right)} = \boxed{0.100 \text{ Kg}} \; (= 100 gm)$

c) Second law (one version of it) states heat never flows spontaneously from cold to hot ... but this is not spontaneous, as the surroundings have to do work (via electrical power) on the system. Entropy of refrigerator + surroundings does not decrease!

d) $\frac{Q}{t} = \frac{kA(\Delta T)}{d} = \frac{\left(220 \frac{J}{s \cdot m \cdot °C}\right)(0.20m \times 0.30m)(20°C - (-10°C))}{(0.5\times10^{-2} m)}$

$= \boxed{79.2 \text{ KW}}$

this is humungously large → that is why your fridge needs to have much better insulation than just thin aluminium walls ...

Problem 7.

The root-mean-square speed of molecules in a certain ideal gas is found to be 900 m/s. There are 10^{21} molecules of this gas in a container with a volume of 100 cm^3. The (absolute) pressure of this gas is found to be 45.1×10^3 Pa.

a) What is the mass of one of the molecules?

b) If the gas is allowed to expanded adiabatically (i.e. with no heat flow in or out of the gas) will the internal energy increase, decrease or stay the same? Explain your in answer in a sentence or using an equation.

a) $v_{rms} = \sqrt{\frac{3KT}{m}}$ $\quad PV = NKT$ (ideal gas) $\rightarrow T = \frac{PV}{NK}$

solve for m, given P, V, N

$$m = \frac{3KT}{(v_{rms})^2} = \frac{3K(PV/NK)}{(v_{rms})^2} = \frac{3PV}{N(v_{rms})^2}$$

$$\left(V = 100\,cm^3 = 100\,cm^3 \times \frac{1\,m^3}{10^6\,cm^3} = 10^{-4}\,m^3\right)$$

$$\therefore m = \frac{3(45\times10^3\,Pa)(10^{-4}\,m^3)}{(10^{21})(900\,m/s)^2} = \boxed{1.67\times10^{-26}\,Kg}$$

b) $\Delta U = Q - W$ (1st law)

$Q = 0$ here (adiabatic) $\therefore \Delta U = -W$

expands $\therefore$ does work $\therefore W =$ positive

$\therefore \Delta U =$ negative $\Rightarrow$ **decreases**

($\therefore$ it cools down)

Problem 8.

Yellow light has a frequency of 5.2×10^{14} Hz.

a) What is its wavelength (in air)?

b) The light is incident on two slits placed one millimeter apart. How far from the slits should a screen be placed such that the distance between the m=0 and m=1 bright fringes seen on the screen is one centimeter?

c) Now suppose that the entire experiment (with the screen at the same location) is performed under water (n=1.33). What would the distance between the m=0 and m=1 bright fringes now be?

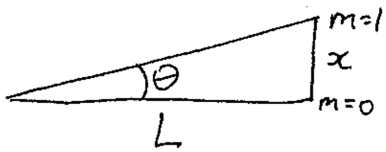
a) $c = \lambda f \quad \therefore \lambda = \frac{c}{f} = \frac{3\times10^{8}\, m/s}{5.2\times10^{14}\, Hz} = 5.77\times10^{-7}\, m = \boxed{577\, nm}$

b) $m=0$ fringe is @ $0°$

$m=1$ fringe $\rightarrow \quad m\lambda = d\sin\theta \qquad \sin\theta = \frac{m\lambda}{d} = \frac{\lambda}{d} = \frac{577\, nm}{10^{-3} m} = 5.77\times10^{-4}$

$\theta = \sin^{-1}(5.77\times10^{-4}) = 0.033°$

, better, $\theta \simeq \sin\theta = 5.77\times10^{-4}$ rad.



$\tan\theta = \frac{x}{L} \quad \therefore L = \frac{x}{\tan\theta} = \frac{10^{-2} m}{\tan(0.033°)} = \boxed{17.3\, m}$

c) $\lambda' = \frac{\lambda}{n} = \frac{577\, nm}{1.33} = 434\, nm$

$\theta = \sin^{-1}\left(\frac{\lambda'}{d}\right) = \sin^{-1}\left(\frac{434\, nm}{10^{-3} m}\right) = 0.025°$

$x = L\tan\theta = (17.3 m)\tan 0.025° = 7.5\, mm = \boxed{0.75\, cm}$

Problem 9.

A doctor examines a mole using a 15.0 cm focal length magnifying glass (convex lens) which she holds 13.5 cm from the mole.

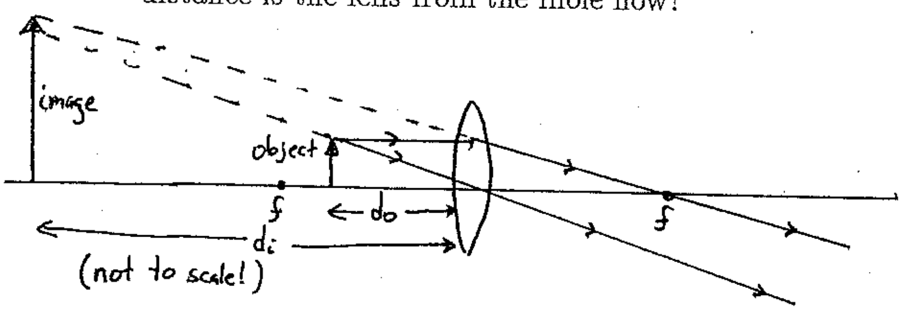
a) Where is the image? (indicate if it is on the object side or the opposite side of the lens, and how far it is from the lens).

b) If the mole has a diameter of 5.00 mm, how big is the image?

c) Is the image inverted or right-side-up?

d) Is the image real or virtual?

e) She now holds the magnifying glass further from the mole, in such a way that she sees a real, inverted image that is 5.00 mm in diameter (same size as the actual mole). What distance is the lens from the mole now?



a) $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

$\therefore \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15cm} - \frac{1}{13.5cm}$

$= -7.41 \times 10^{-3} cm^{-1}$

$\therefore$ $d_i = -135 cm$ ← on object side of lens

b) $m = -\frac{d_i}{d_o} = -\frac{(-135 cm)}{13.5 cm} = +10 = \frac{h_i}{h_o}$

$\therefore h_i = 10 h_o = 10(5mm) =$ 50mm in diameter

c) image is right side up
($m = +10$, h_i = positive ; see sketch)

d) image is virtual
(on object side of lens)

e) $m = -1 = -\frac{d_i}{d_o}$ $\therefore d_i = d_o$

$\therefore \frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{d_o} + \frac{1}{d_o} = \frac{1}{f}$

$= \frac{2}{d_o} = \frac{1}{f}$ $\therefore d_o = 2f$

$= 2(15.0 cm)$

$d_o = 30.0 cm$