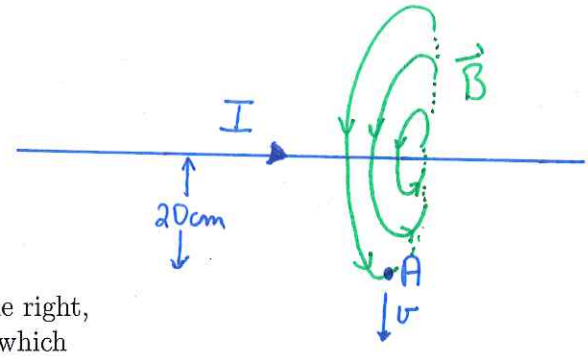


Problem 1. [20 points]

A long, straight wire carries a current to the right, as shown. The magnetic field at a location A, which is 20 cm below the wire, is measured to be 2×10^{-4} Tesla.

a) [5 points] What is the *direction* of the magnetic field at location A?

b) [7 points] What is the magnitude of the current I?

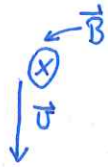
c) [8 points] An electron is located at A, and is moving downward with a speed $v = 300$ m/s. What is the magnitude *and* direction of the magnetic force on that electron?

a) Right Hand rule : $\vec{B}$ @ location A is $\boxed{\text{into the page}}$

b) $B = \dfrac{\mu_0 I}{2\pi r}$ for a long, straight wire

$$\therefore I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.20\,m)(2\times10^{-4}\,T)}{4\pi\times10^{-7}\,\frac{Tm}{A}} = \boxed{200\,A}$$

c) $F = |q| v B \sin\theta \qquad \theta = 90° \therefore \sin\theta = 1$



$$= (1.6\times10^{-19}\,C)\left(300\,\frac{m}{s}\right)(2\times10^{-4}\,T)$$

$$= \boxed{9.6\times10^{-21}\,N}$$

Direction : Right Hand rule says force on a positive particle would be to the right.
But electron has negative charge
$\therefore$ opposite direction for force
$\therefore$ force is to the $\boxed{\text{left}}$

Problem 2. [25 points]

a) [5 points] An electromagnetic wave has a peak electric field strength of 0.1 V/m. What is the peak magnetic field strength in this wave?

b) [5 points] What is the intensity of this wave?

c) [5 points] This wave is incident on your eye. How much energy is carried in one minute by this wave through the 4 mm diameter pupil of your eye?

d) [5 points] The wave has a frequency of 4.5×10^{14} Hz. What is the wavelength of the light in vacuum?

e) [5 points] The vitreus humour (the gel that makes up your eyeball) has an index of refraction of 1.337. What is the wavelength of this light when it is inside your eyeball?

a) $E_0 = 0.1\ V/m \qquad c = \frac{E}{B} \quad \therefore \quad B_0 = \frac{E_0}{c} = \frac{0.1\ V/m}{3\times10^8\ m/s} = \boxed{3.33\times10^{-10}\ T}$

b) $I_{ave} = \frac{1}{2} c \varepsilon_0 E_0^2 = \frac{1}{2}(3\times10^8\ m/s)(8.85\times10^{-12}\ \frac{C^2}{Nm^2})(0.1\ \frac{V}{m})^2 = \boxed{1.33\times10^{-5}\ W/m^2}$

c) area of pupil $= \pi r^2 = \pi(d/2)^2 = \pi\left(\frac{4\times10^{-3}\ m}{2}\right)^2 = 1.26\times10^{-5}\ m^2$

$I = P/A \quad \therefore \quad P = IA = (1.33\times10^{-5}\ W/m^2)(1.26\times10^{-5}\ m^2) = 1.67\times10^{-10}\ W$

$P = E/t \quad \therefore \quad E = Pt = (1.67\times10^{-10}\ W)(60\ s) = \boxed{1.00\times10^{-8}\ J}$

d) $c = \lambda f \quad \therefore \quad \lambda = c/f = \frac{3.00\times10^8\ m/s}{4.5\times10^{14}\ Hz} = 6.67\times10^{-7}\ m = \boxed{667\ nm}$

e) $n = c/v \quad \therefore \quad v_{eyeball} = c/n = \frac{3\times10^8\ m/s}{1.337} = 2.24\times10^8\ m/s$

f is unchanged by medium

$v = \lambda f \quad \therefore \quad \lambda = v/f = \frac{2.24\times10^8\ m/s}{4.5\times10^{14}\ s^{-1}} = \boxed{499\ nm}$

Problem 3. [30 points]

A veterinarian uses a single convex lens as a magnifiying glass to examine a tick on a dog's ear. The lens has a focal length of 10 cm. The tick is 2.0 mm in length, but the image of the tick as seen through the lens is 5.0 mm in length. The image is upright.

a) [5 points] What is the power of the lens?

b) [8 points] How far away from the tick is the vet holding the lens?

c) [7 points] How far from the lens is the image of the tick?

d) [5 points] Is the image real or virtual?

e) [5 points] Draw a ray-diagram sketch showing the object, the image, the lens, and at least two rays. It should be *roughly* to scale.

a) $P = 1/f = 1/0.10\,m = 10\,m^{-1} = \boxed{10 \text{ Diopters}}$

b) $m = \frac{h_i}{h_o} = \frac{5.0\,mm}{2.0\,mm} = +2.5$ (upright)

$m = -\frac{d_i}{d_o} = 2.5 \quad \therefore \quad d_i = -2.5 d_o$

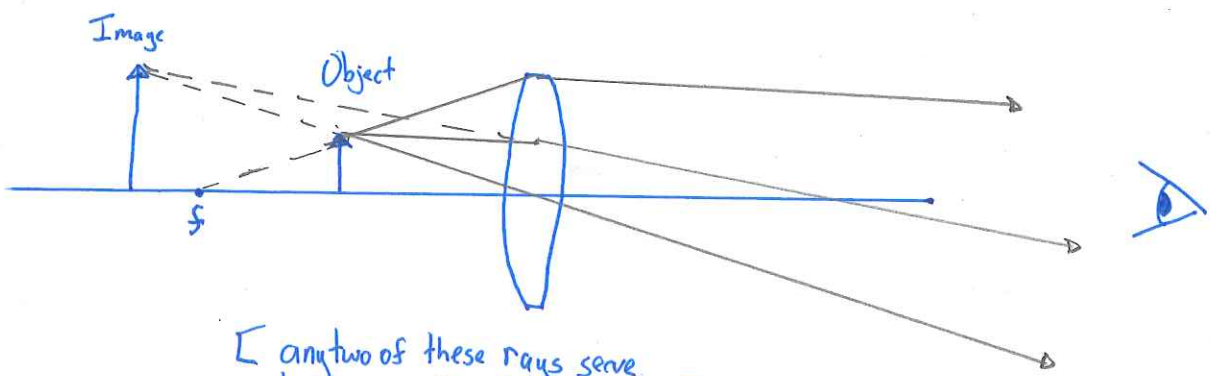
$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \therefore \quad \frac{1}{d_o} + \frac{1}{-2.5 d_o} = \frac{1}{f} \quad \therefore \quad \frac{-2.5+1}{-2.5 d_o} = \frac{1}{f}$

$d_o = \frac{f(2.5-1)}{2.5} = (10\,cm)\frac{1.5}{2.5} = \boxed{6.0\,cm}$

c) $d_i = -2.5 d_o = -2.5(6.0\,cm) = -15\,cm$

$\therefore$ $\boxed{15\,cm \text{ from lens}}$ (negative sign means is on same side of lens as object)

d) d_i = negative $\therefore$ $\boxed{\text{virtual}}$



[any two of these rays serve to locate the (virtual) image]

Problem 4. [25 points]

The magnet of an MRI scanner is a solenoid, with a length of 200 cm, with 5000 turns of wire, which carries a current of 400 Amps.

a) [8 points] What is the strength of the magnetic field in the solenoid?

b) [12 points] A patient has an eyebrow ring which has a diameter of 1 cm. The patient's head enters the solenoid, and so her head goes from a region of zero magnetic field to the full field of the solenoid in half a second. What is the electromotive force induced in the eyebrow ring? (You should assume that the plane of the ring is perpendicular to the magnetic field).

c) [5 points] Assume you are looking at the ring, and you are looking in the direction of the magnetic field (*i.e.* the magnetic field points from you toward the ring). From your perspective, will the induced current in the ring be clockwise or counterclockwise? Explain your reasoning, don't just give the answer.

a) Solenoid : $B = \mu_0 n I = \mu_0 \frac{N}{L} I = (4\pi \times 10^{-7} \frac{T \cdot m}{A})(\frac{5000}{2.00m})(400A) = \boxed{1.26\ T}$

$n = \frac{\text{turns}}{\text{length}}$

b) $A = \pi r^2 = \frac{\pi d^2}{4} = \frac{\pi}{4}(1.0 \times 10^{-2} m)^2 = 7.85 \times 10^{-5} m^2$

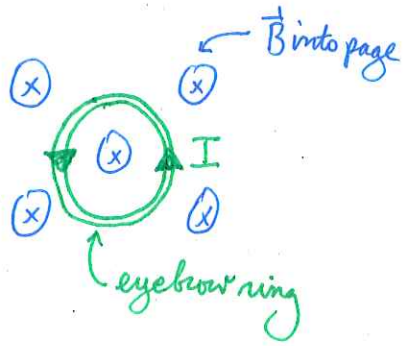
$\phi_{initial} = 0$ $\quad \phi_{final} = BA\cos\theta = (1.26T)(7.85 \times 10^{-5} m^2)\cos 0° = 9.89 \times 10^{-5}$ Wb

(N=1 for eyebrow ring)

Faraday's law : $|\mathcal{E}| = \frac{\Delta\phi}{\Delta t} = \frac{\phi_{final} - \phi_{initial}}{0.5s} = \frac{9.89 \times 10^{-5} Wb}{0.5s} = \boxed{1.98 \times 10^{-4} V}$

(0.198 mV) ∴ tiny

c)



Flux through the ring (due to solenoid's field) is into page & increasing. Lenz's law says induced EMF is to oppose the change ∴ to produced induced $\vec{B}$ out of page. From right hand rule, would need a counterclockwise current to create $\vec{B}$ inside ring which points out of page ∴ I = $\boxed{\text{Counterclockwise}}$