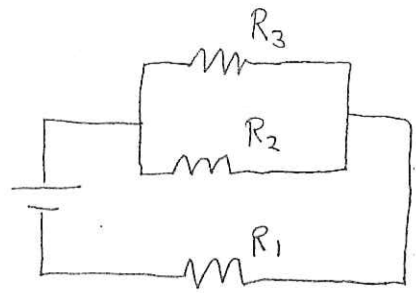
See: 02/29/16 lecture
& Last year's Test, problem #2



Problem 1. [25 points]

Consider the circuit shown, where V = 10 V, $R_1 = 3$ kΩ, $R_2 = 10$ kΩ, and $R_3 = 10$ kΩ.

a) [7 points] What is the total equivalent resistance of the three resistors?

b) [8 points] What is the current through each resistor?

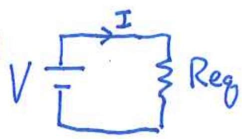
c) [5 points] How much energy is dissipated as heat in resistor R_1 in 10 seconds?

d) [5 points] Assume that the resistors are actually the filaments of incandescent light bulbs. If R_2 is disconnected from the circuit, will the R_1 bulb get brighter or dimmer? Explain your reasoning.

a) $\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{10K\Omega} + \frac{1}{10K\Omega} = \frac{1}{5K\Omega} \therefore R_p = 5\ K\Omega$

this is in series with R_1 $\therefore$ $R_{eq} = R_1 + R_p = 3K\Omega + 5K\Omega = \boxed{8K\Omega}$

b)



$I = \frac{V}{R_{eq}} = \frac{10V}{8K\Omega} = 1.25\times10^{-3}A$ $\boxed{I_1 = I = 1.25\times10^{-3}A}$

$V_1 = I_1 R_1 = (1.25\times10^{-3}A)(3\times10^3\Omega) = 3.75\ V$

$\therefore V_2 = V_3 = (V - V_1) = 10V - 3.75V = 6.25V$

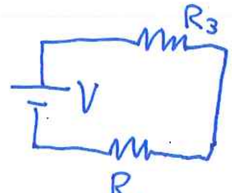
$I_2 = \frac{V_2}{R_2} = \frac{6.25V}{10^4\Omega} = \boxed{6.25\times10^{-4}A}$

$I_3 = \frac{V_3}{R_3} =$ " $=$ "

(or: since $R_2 = R_3$ must have $I_2 = I_3$
& $I_2 + I_3 = I_1$ $\therefore$ $I_2 = I_3 = I_1/2$ ...)

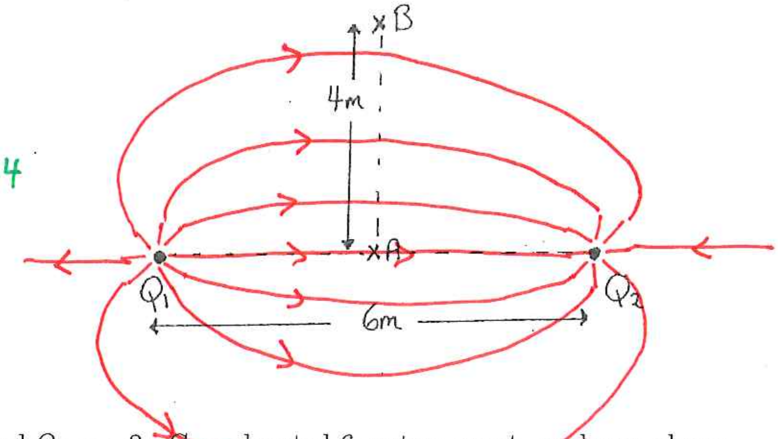
c) $P_1 = I_1^2 R_1 = (1.25\times10^{-3}A)^2(3\times10^3\Omega) = 4.69\times10^{-3}W$ $E = Pt = (4.69\times10^{-3}W)(10s) = \boxed{4.69\times10^{-2}J}$

d) Circuit is now:



Since R_2 is removed, the resistance of what was the parallel combination has increased (is now 10 KΩ, it was 5 KΩ) $\therefore$ total R_{eq} has increased (it is now $R_2 + R_3$ = 13 KΩ instead of 8KΩ) Thus the current $I = I_1$ has decreased (V is the same) → therefore the power has decreased, thus the bulb is **dimmer**

See: 02/10/16 lecture
& Last year's test, problem #4



Problem 2. [25 points]

Two charges, $Q_1 = +2\ \mu\text{C}$ and $Q_2 = -2\ \mu\text{C}$ are located 6 meters apart, as shown above. Location A is midway between the charges, and location B is 4 meters above the midpoint.

a) [6 points] Find the electric field (magnitude and direction) at location A.

b) [5 points] What would be the force (magnitude & direction) on an electron if it was located at A?

c) [4 points] What is the direction (don't worry about the magnitude) of the electric field at location B?

d) [6 points] What is the electric potential at location B?

e) [4 points] Sketch at least 8 electric fields lines generated by these charges.

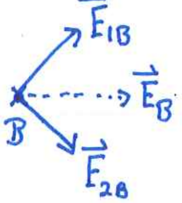
a) $E_1 = \frac{K|Q_1|}{r_1^2} = \frac{(9\times10^9\ \frac{Nm^2}{C^2})(2\times10^{-6}C)}{(3m)^2} = 2000\ N/C$; to the right

$E_2 = \frac{K|Q_2|}{r_2^2} = \frac{(9\times10^9\ \frac{Nm^2}{C^2})(2\times10^{-6}C)}{(3m)^2} =$ " "

Total: $\vec{E}_A = \vec{E}_1 + \vec{E}_2$
$= \boxed{4000\ N/C, \text{ to the right}}$

b) $|F| = |q_e| E_A = (1.6\times10^{-19}C)(4000\ \frac{N}{C}) = \boxed{6.4\times10^{-16}N}$ $\boxed{\text{to the left}}$ ($\vec{F} = q\vec{E}$ & q = negative)

c)



vertical components cancel
horizontal components add
$\therefore$ $\boxed{\text{to the right}}$

d) $V_B = \frac{KQ_1}{r_1} + \frac{KQ_2}{r_2}$ $\quad r_1 = r_2 = 5m$ here $\quad \therefore V_B = \frac{K}{(5m)}[Q_1 + Q_2] = \frac{K}{(5m)}[2\mu C - 2\mu C]$

$= \boxed{0V}$

e) see sketch above;
- lines run from + to − charges ; equal number leave as arrive (equal charges) ; lines don't cross ...

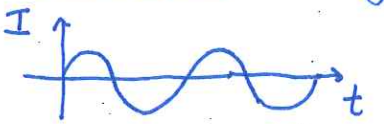
3. [20 points] (these are three unrelated problems)

a) [7 points] You are toasting your breakfast bagel using a 3.2 amp DC current entering your toaster. The current is on for 10 seconds. How many electrons go through the toaster during this time? If it was, instead, a 3.2 amp AC current, how many electrons go through the toaster?

DC: $I = \frac{\Delta Q}{\Delta t}$ $\therefore$ $\Delta Q = I \Delta t = (3.2A)(10s) = 32C$

$Q = n|q_e|$ where n = # electrons $\therefore$ $n = \frac{Q}{|q_e|} = \frac{32C}{1.6\times10^{-19}C} = \boxed{2\times10^{20}}$

AC: $\simeq$ Zero electrons → charge sloshes back and forth, but no net charge moves through toaster



b) [7 points] A 3 μF capacitor is charged to a voltage of 20 V. If it is then discharged through a 100 MΩ resistor, how long does it take for the voltage on the capacitor to fall to 1 V?

$\tau = RC = (100\times10^{6}\Omega)(3\times10^{-6}F) = 300s$

$V(t) = V_0 e^{-t/\tau}$

$1V = (20V)e^{-t/\tau}$ $\therefore$ $e^{-t/\tau} = \frac{1}{20}$ $\ln\left(\frac{1}{20}\right) = -t/\tau$

See: Practice problems for Test 2, #9 & Last year's test Problem #1 d)

$\therefore t = -\tau \ln(1/20)$

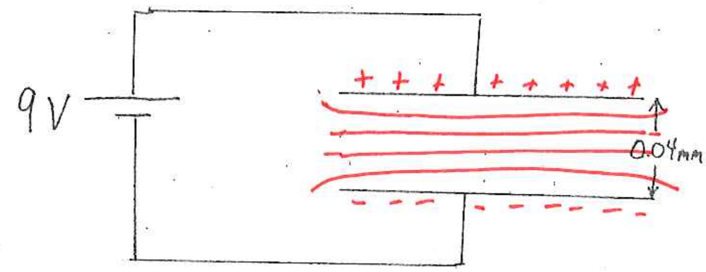
$= -(300s)\ln(0.05)$

$= \boxed{899s}$ $= 15 min.$

c) [6 points] You have connected a Galvanometer in series with a 10 MΩ resistor. Is the resulting device useful as an Ammeter or as a Voltmeter?

Voltmeters are made using galvanometers in series with a large resistance.

Problem 4. [30 points]



The plates of a parallel-plate capacitor each have an area of 16 cm^2, and they are separated by 0.04 mm. It is connected to a 9 V battery as shown.

a) [5 points] What is the magnitude of the charge on each plate?

b) [5 points] On the diagram, draw several equipotential lines in the region between the capacitor plates.

c) [5 points] Which way (up, down, left, or right) does the electric field between the plates point?

d) [5 points] What is the magnitude of the electric field between the plates?

e) [5 points] The battery is disconnected from the circuit (without allowing the capacitor to discharge). Now, a sheet of paraffin (dielectric constant = 2.2) is inserted between the plates; it fills the space completely. What is the new charge and the new voltage on the capacitor?

f) [5 points] Prof. Armstrong takes a much larger capacitor, $C = 100\ \mu F$, and charges it using a 250 V power supply. He then disconnects it from the power supply, and connects a metal screwdriver across the plates. All the stored energy is discharged in 10^{-6} s with a loud bang. What is the power (in watts) of this discharge?

a) $C = \frac{\kappa \varepsilon_0 A}{d}$ $\kappa = 1$ for air

$C = (1)(8.85\times10^{-12} F/m)\left(\frac{16\times10^{-4} m^2}{0.04\times10^{-3} m}\right) = 3.54\times10^{-10} F = 354\ pF$

$Q = CV = (3.54\times10^{-10} F)(9V) = \boxed{3.19\times10^{-9} C}$

b) see diagram; equally spaced parallel lines

c) $\vec{E}$ points $\boxed{\text{down}}$ (away from positive charges)

d) Uniform field : $V = Ed \therefore E = \frac{V}{d} = \frac{9V}{0.04\times10^{-3} m} = \boxed{2.25\times10^{5} V/m} = 225\ kV/m$

e) $C_{new} = \frac{\kappa \varepsilon_0 A}{d} = \kappa C = (2.2)(3.54\times10^{-10} F) = 7.79\times10^{-10} F$

$Q_{new} = Q$ (no discharge!) (disconnected from battery – nowhere for charge to go) $= \boxed{3.19\times10^{-9} C}$

$Q_{new} = C_{new} V_{new} \quad \therefore V_{new} = \frac{Q_{new}}{C_{new}} = \frac{3.19\times10^{-9} C}{7.79\times10^{-10} F} = \boxed{4.09 V}$

f) $E_{cap} = \frac{1}{2} C V^2 = \frac{1}{2}(100\times10^{-6} F)(250 V)^2 = 3.12\ J$

$\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{3.12 J}{10^{-6} s} = \boxed{3.12\times10^{6} W}$

3 Megawatts!