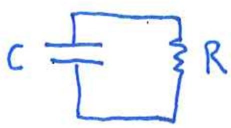
1. A cardiac defibrillator delivers 400 Joules of energy by discharging a capacitor which is initially at 10,000 Volts

(a)(5) What is the capacitance of the capacitor?
(b)(5) Assume that the defibrillator discharges through a resistance of 1500 Ohms (the patient's body). How long is it before the voltage drops to 1/e of its initial value?
(c)(5) What is the initial current through the patient's body?
(d)(5) What is the defibrillator voltage one-tenth of a second after the discharge starts?
(e)(5) A capacitor discharged in class had wires sticking up, and the discharge was made by touching a piece of metal to both leads. Defibrillators use paddles, with a much larger surface area than the wires. Why? (Be qualitative)

a) $E_{cap} = \frac{1}{2}CV^2 \quad \therefore C = \frac{2E_{cap}}{V^2} = \frac{2(400\,J)}{(10^4 V)^2} = 8\times10^{-6}F = \boxed{8\mu F}$

b) $\tau = RC = (1500\Omega)(8\times10^{-6}F) = 0.012s \qquad V = V_0 e^{-t/\tau} \quad \therefore \frac{V}{V_0} = \frac{1}{e}$ when $\tau = t$ $\therefore t = \boxed{0.012s}$

c)

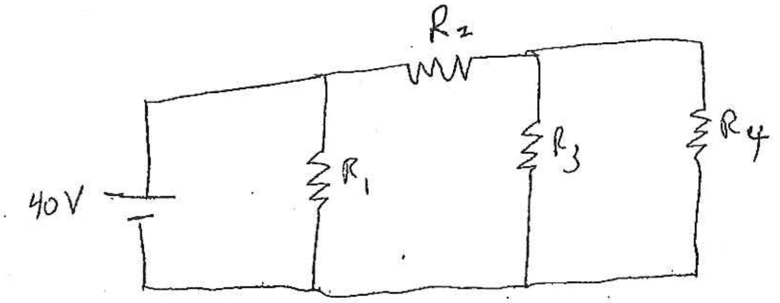


$V_C = V_R \qquad V(t=0) = V_0 e^{-0/\tau} = V_0$

$V_C = IR \quad \therefore I = V/R = V_0/R = \frac{10^4 V}{1500\Omega} = \boxed{6.67\,A}$

d) $V = V_0 e^{-t/\tau} = (10^4 V) e^{-(0.1s)/(0.012s)} = \boxed{2.40V}$

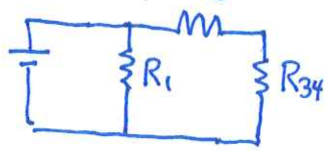
e) Larger surface area → volume of tissue affected is greater ∴ the I^2R heating is distributed over larger volume ∴ reduced temperature rise (due to $Q = mc\Delta T$) ∴ less tissue damage.



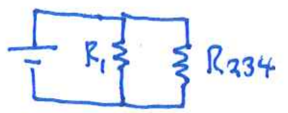
2. Four resistors are connected to a 40V battery as shown. $R_1 = 20$ ohms, $R_2 = 8$ ohms, $R_3 = 3$ ohms and $R_4 = 6$ ohms.

(a)(8) Find the equivalent resistance and the current coming out of the battery.
(b)(9) Find the current through, voltage across and power generated by each resistor.
(c)(8) Suppose R_1 burns out. Explain why R_2, R_3 and R_4 do not get brighter or dimmer. (Note: you can recalculate them, or use a more qualitative argument).

a) R_3 in parallel with R_4 $\therefore$ $\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{3\Omega} + \frac{1}{6\Omega}$ $\therefore$ $R_{34} = 2\Omega$

$\equiv$ 

$\therefore$ $R_{234} = R_2 + R_{34} = 8\Omega + 2\Omega = 10\Omega$
(series combo)

$\equiv$ 

$\therefore$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_{234}} = \frac{1}{20\Omega} + \frac{1}{10\Omega}$ $\therefore$ $R_{eq} = \boxed{\frac{20}{3}\Omega}$

$I = \frac{V}{R_{eq}} = \frac{40V}{20/3\,\Omega} = \boxed{6A}$

b) $V_1 = V = \boxed{40V}$ $\therefore$ $I_1 = \frac{40V}{20\Omega} = \boxed{2A}$ $P_1 = I_1 V_1 = \boxed{80W}$

6A from Battery, 2A through R_1 $\therefore$ $6A - 2A = 4A$ through R_2 $\boxed{I_2 = 4A}$

$\therefore V_2 = I_2 R_2 = (4A)(8\Omega) = \boxed{32V}$
$P_2 = I_2 V_2 = (4A)(32V) = \boxed{128W}$

$V_3 = V_4$ (in parallel) $= V_B - V_2 = 40V - 32V = \boxed{8V = V_3 = V_4}$

$I_3 = \frac{V_3}{R_3} = \frac{8V}{3\Omega} = \boxed{\frac{8}{3}A}$ $I_4 = \frac{8V}{6\Omega} = \frac{8}{6}A = \boxed{\frac{4}{3}A}$

$P_3 = I_3 V_3 = \left(\frac{8}{3}A\right)(8V) = \boxed{\frac{64}{3}W}$ $P_4 = I_4 V_4 = \left(\frac{4}{3}A\right)(8V) = \boxed{\frac{32}{3}W}$

Check: $P_B = IV = (6A)(40V) = 240W$ ✓
$= P_1 + P_2 + P_3 + P_4 = 240W$

c) R_1 is in parallel with Battery $\therefore$ removing it will still leave 40V across the combination of the remaining 3 resistors → since their resistances are unchanged & the voltage across them is also unchanged, their currents & $\therefore$ their powers ($\therefore$ brightness if they are the filaments of light bulbs) are also unchanged.

3. These are unrelated short answer questions.

(a)(8)You have a hollow metal sphere. You put a positive charge on the sphere.

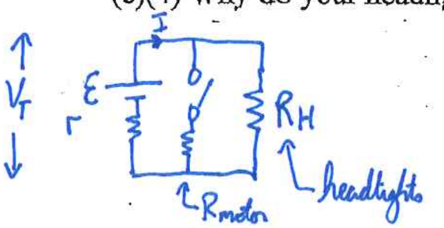
Just outside the surface is the direction of the electric field (a) away from the center of the sphere, (b) towards the center of the sphere or (c) zero? Answer here: (a)
Just inside the surface is the direction of the electric field (a) away from the center of the sphere, (b) towards the center of the sphere or (c) zero? Answer here: (c)

(Field points away from + charges & is zero inside a conductor)

(b)(4) Three identical resistors are connected in series. Their equivalent resistance is 12 ohms. If they are disconnected and reattached in parallel, what is their equivalent resistance? $R_s = R_1 + R_2 + R_3 = 12\Omega$ $R_1 = R_2 = R_3$ $\therefore$ $3R = 12\Omega$ $R = 4\Omega$ for each one

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{4\Omega} \quad \therefore \quad \boxed{R_p = \frac{4}{3}\Omega}$$

(c)(4) Why do your headlights dim when you start your car engine?



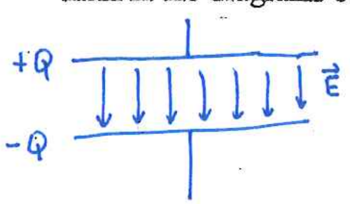
V_T: terminal voltage $= \mathcal{E} - Ir$

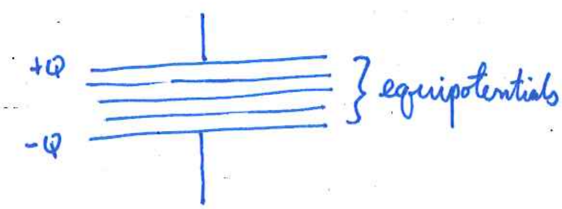
Start motor ; I increases $\therefore$ V_T decreases

$\therefore$ less current to headlights

$\therefore$ less power in " "

(d)(5) Draw electric field lines and equipotential lines in a parallel plate capacitor. Draw them in the diagrams below.





(e)(4) A capacitor is charged to a potential of 10 volts. An electron is put at the negative plate and released. It moves to the positive plate. When it reaches the positive plate, what is its kinetic energy?

$$PE = q\Delta V = 10eV \quad \therefore \quad \boxed{KE = 10eV}$$

$$\left[= (1.6\times10^{-19}C)(10V) = 1.6\times10^{-18}J = 10eV \right]$$

4. In the above diagram, the charges are 6 cm apart. $Q_1 = -1$ microcoulomb and $Q_2 = +2$ microcoulombs. Point A is midway between them, and point B is 3 cm to the left of Q_1.
(a)(10) Find the potential and the electric field at point A.
(b)(10) Find the potential and the electric field at point B.
(c)(5) Draw electric field lines. You should have at least eight lines total.

a) $V = \frac{KQ}{r}$: scalar, not vector ∴ just adds (no direction)

$$V_A = \frac{KQ_1}{r_1} + \frac{KQ_2}{r_2} = \frac{K}{r}(Q_1 + Q_2) = \frac{9\times10^9 \frac{Nm^2}{C^2}}{3\times10^{-2}m}\left(-1\times10^{-6}C + 2\times10^{-6}C\right) = \boxed{3\times10^5 V}$$

$r_1 = r_2 = 3cm$

$|\vec{E}| = \frac{K|Q|}{r^2}$; $\vec{E}$ is vector (has direction). Call $+x$ direction to the right.

$$\vec{E}_A = -\frac{K|Q_1|}{r^2} - \frac{K|Q_2|}{r^2} = -\frac{K}{r^2}(|Q_1| + |Q_2|) = -\frac{\left(9\times10^9 \frac{Nm^2}{C^2}\right)}{(3\times10^{-2}m)^2}\left(3\times10^{-6}C\right) = \boxed{-3\times10^7 N/C}$$

(negative since both individual fields to the left) (i.e. to the left)

b) $$V_B = \frac{KQ_1}{r_1} + \frac{KQ_2}{r_2} = K\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right) = 9\times10^9 \frac{Nm^2}{C^2}\left[\frac{-1\times10^{-6}C}{3\times10^{-2}m} + \frac{2\times10^{-6}C}{9\times10^{-2}m}\right] = \boxed{-1\times10^5 V}$$

$$\vec{E}_B = +\frac{K|Q_1|}{r_1^2} - \frac{K|Q_2|}{r_2^2} = K\left(\frac{|Q_1|}{r_1^2} - \frac{|Q_2|}{r_2^2}\right) = 9\times10^9 \frac{Nm^2}{C^2}\left[\frac{1\times10^{-6}C}{(3\times10^{-2}m)^2} - \frac{2\times10^{-6}C}{(9\times10^{-2}m)^2}\right] = \boxed{7.78\times10^6 N/C}$$

(again, + is to the right) i.e. to the right

c)

$\vec{E}$ Lines : away from Q_2
: toward Q_1
: twice as many lines Q_2 as end on Q_1