Problem 1.

A dentist using a dental drill brings it from rest to maximum operating speed of 400,000 rpm (rotations per minute) in 3.0 s. Assume that the drill accelerates at a constant rate during this time.

a) What is the angular acceleration of the drill?

b) Find the number of revolutions the drill bit makes during the 3.0 time interval.

a) $$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\left(4\times10^{5}\frac{\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}} - 0\right)}{3\,\text{s}} = \boxed{1.40\times10^{4}\ \text{rad}/\text{s}^2}$$

b) $$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$\omega_0 = 0$

$\theta_0 = 0$

$$\therefore\ \theta = \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2}\left(1.40\times10^{4}\ \text{rad}/\text{s}^2\right)(3\,\text{s})^2 = 6.30\times10^{4}\ \text{rad}$$

$$6.30\times10^{4}\ \text{rad} \times \frac{1\,\text{rev}}{(2\pi\,\text{rad})} = \boxed{1.00\times10^{4}\ \text{revolutions}}$$

Note: this was problem #6 on Homework #9

Problem 2.

Professor Archimedes has a mass of 65 kg. When he is weighed while completely submerged in ordinary water ($\rho = 10^3$ kg/m^3), his apparent weight is only 10 N.

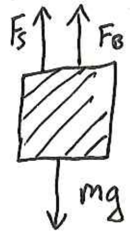
a) What is the bouyant force exerted on him by the water?

b) What is his volume?

c) If he was immersed in sea water, with a specific gravity of 1.03, would he float or sink? (don't just give the answer, you need to show your calculation).

a) Apparent weight = force on Archimedes due to scales = F_s

Free Body Diagram on ancient Greek :



F_B = Bouyant Force

mg = Actual Weight

$\vec{F}_{net} = 0 \quad \therefore \quad F_s + F_B - mg = 0$

$\therefore \; F_B = mg - F_s$

$= (65\,\text{Kg})(9.8\,\text{m/s}^2) - 10\,\text{N} = 637\,\text{N} - 10\,\text{N} = \boxed{627\,\text{N}}$

b) $F_B = W_{disp}$ (Archimedes's Principle)

$= \rho_{H_2O} V g \quad \therefore \quad V = \dfrac{F_B}{\rho_{H_2O}\, g} = \dfrac{627\,\text{N}}{(10^3\,\frac{\text{Kg}}{\text{m}^3})(9.8\,\text{m/s}^2)} = \boxed{6.40 \times 10^{-2}\,\text{m}^3}$

c) calculate F_B in sea water : $F_B' = \rho_{sea} V g = (1.03)(10^3\,\frac{\text{Kg}}{\text{m}^3})(6.40 \times 10^{-2}\,\text{m}^3)(9.8\,\text{m/s}^2)$

$\rho_{sea} = (1.03)\rho_{H_2O}$ $\qquad = 646\,\text{N}$

(specific gravity = 1.03)

$\therefore \; F_B' > mg \quad \therefore$ will float

$\boxed{\text{or}}$: $\rho_{Archimedes} = \dfrac{m}{V} = \dfrac{65\,\text{Kg}}{6.40 \times 10^{-2}\,\text{m}^3} = 1.016 \times 10^3\,\text{Kg/m}^3$

$< \rho_{sea} = 1.030 \times 10^3\,\text{Kg/m}^3$

Note: Compare to problem 5 on Homework 10 ; essentially same ...

$\therefore$ his density is less than that of sea water

$\therefore$ he floats (barely ... excuse the pun)

Problem 3.

In class, Dr. Chiesa sat on a rotatable stool with two 4-kg weights held in his outstretched hands. Assume that Chiesa (by himself) has a moment of inertia of 3.0 kg·m^2, and holds the weights at a distance of 80 cm from his center. Treat the 4-kg weights as point masses ($I = Mr^2$) and ignore the mass of his arms. He sits on the stool and spins at a rate of once every three seconds.

a) What is the total moment of inertia of Chiesa plus the weights?

b) What is the initial angular momentum?

c) He then brings the weights into his chest (so they are on the axis of rotation). What is his new angular velocity?

d) How much work did Chiesa do in order to bring the weights to his chest?

d) Friction then applies a torque of 2.4 N·m. How long does it take for Dr. Chiesa to stop spinning, from the moment the friction was introduced?

a) $I_i = I_{chiesa} + 2 \cdot I_{weight} = 3.0\,\text{Kgm}^2 + 2 \times (4\,\text{Kg})(0.80\,\text{m})^2 = \boxed{8.12\,\text{Kgm}^2}$

$I_{weight} = mr^2$

b) $L_i = I_i \omega_i = (8.12\,\text{Kgm}^2)(2.09\,\text{rad/s}) = \boxed{17.0\,\text{Kgm}^2/\text{s}}$

$\omega_i = 2\pi f = 2\pi\left(\frac{1}{3}\,\text{s}\right) = \frac{2}{3}\pi\,\text{rad/s} = 2.09\,\frac{\text{rad}}{\text{s}}$

c) $L_f = L_i$ (angular momentum conserved)

$I_f \omega_f = I_i \omega_i$ $\qquad \omega_f = \frac{I_i \omega_i}{I_f} = \frac{L_i}{I_f} = \frac{17.0\,\text{Kgm}^2/\text{s}}{3\,\text{Kgm}^2} = \boxed{5.67\,\text{rad/s}}$

$I_f = I_{chiesa}$?

(weights are @ $r = 0$ now)

d) $W_{net} = \Delta KE = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_i \omega_i^2 = \frac{1}{2}(3\,\text{Kgm}^2)\left(5.67\,\frac{\text{rad}}{\text{s}}\right)^2 - \frac{1}{2}(8.12\,\text{Kgm}^2)(2.09\,\text{rad/s})^2$

$= 48.2\,\text{J} - 17.7\,\text{J} = \boxed{30.5\,\text{J}}$

Chiesa used energy (chemical) to pull weights towards him.

e) $\tau = I\alpha$

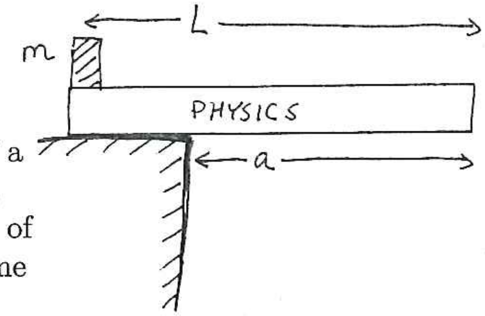
Note: done most of this in-class on 10/28/13

$\alpha = \frac{\tau}{I} = \frac{-2.4\,\text{Nm}}{3\,\text{Kgm}^2} = -0.80\,\text{rad/s}^2$ (sign reflects slowing of rotation)

$\alpha = \frac{\Delta\omega}{\Delta t} \quad \therefore \Delta t = \frac{\Delta\omega}{\alpha} = \frac{0 - 5.67\,\text{rad/s}}{-0.80\,\text{rad/s}^2} = \boxed{7.09\,\text{s}}$

OR: use $\tau = \frac{\Delta L}{\Delta t} \quad \therefore \Delta t = \frac{\Delta L}{\tau} \ldots$

Problem 4.



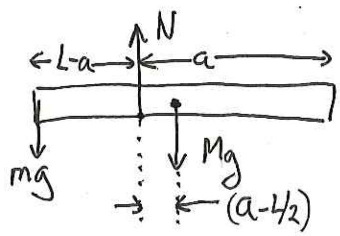
A hardcover physics textbook is balanced at the edge of a table, extending out a distance $a = 20$ cm beyond the edge, and has a mass m sitting on the book, as shown. The mass of the book is $M = 0.8$ kg, and its length is $L = 30$ cm. Assume the center-of-mass of the book is halfway along its length.

a) What is the minimum mass m needed in order to keep the book balanced like this?

b) What is the normal force that the table exerts on the book?

c) Would your answer to part a) be larger, smaller, or the same, if the experiment took place on the moon ($g_{\text{moon}} \approx g/6$)? Explain.

a) Static Equilibrium
FBD on book:



when book is just balanced (a "tipping point"), Normal force table exerts must be @ edge of table.

$\tau_{net} = 0$; use edge of table as pivot point ; call CW as positive for rotations

$$\tau_{net} = -mg(L-a) + Mg\left(a - \frac{L}{2}\right) = 0$$

$$m = \frac{Mg(a - L/2)}{g(L-a)} = \frac{M(a-L/2)}{L-a} = \frac{(0.8\,\text{Kg})\left(0.20 - \frac{0.30}{2}\right)m}{(0.30m - 0.20m)} = \boxed{0.4\ \text{Kg}}$$

b) $F_{net} = 0 = N - mg - Mg = 0$

$$\therefore N = mg + Mg = (m+M)g = (0.4\,\text{Kg} + 0.8\,\text{Kg})(9.8\,\text{m/s}^2) = \boxed{11.8\ \text{N}}$$

c) in the expression for "m" we derived in part a), the "g" cancels out!

$$m = \frac{M(a - L/2)}{(L-a)}$$ $\therefore$ $\boxed{\text{same answer}}$ on moon ... or any body with gravity...

Note: this exact problem done in class on 10/21/13