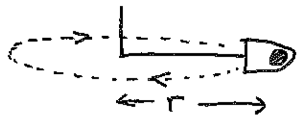
# Problem 1.

A sample of blood is placed in a centrifuge of radius 15 centimeters. The mass of a red corpuscle is $3.0 \times 10^{-16}$ kg, and the magnitude of the force required to make it settle out of the plasma is $4.0 \times 10^{-11}$ N.

a) What must the speed of the sample be?
b) How many revolutions per second must the centrifuge be operated at to obtain this speed?
c) The entire sample has a mass of 0.10 grams. What is the force on the bottom of the test tube?




$m = 3.0 \times 10^{-16}$ Kg
$r = 0.15 m$
$F = 4.0 \times 10^{-11} N$

a) $F = \frac{mv^2}{r} \quad \therefore \quad v = \left(\frac{Fr}{m}\right)^{1/2} = \boxed{141\ m/s}$

b) $v = r\omega \quad \therefore \quad \omega = \frac{v}{r} = \frac{141\ m/s}{0.15 m} = 943\ rad/s$

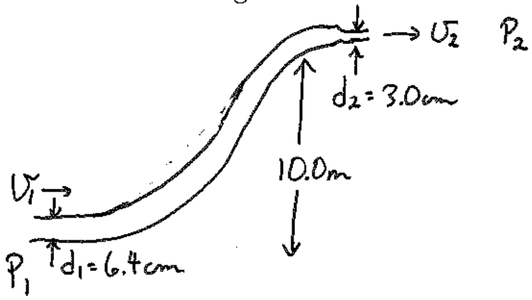
$\left(943\ \frac{rad}{s}\right) \times \left(\frac{1\ revolution}{2\pi\ rad}\right) = \boxed{150\ revs/s}$

c) $F = \frac{mv^2}{r} = \frac{(0.1 \times 10^{-3}\ Kg)(141\ m/s)^2}{0.15 m} = \boxed{13.3 N}$

## Problem 2.

A fire hose has an inside diameter of 6.4 cm, and carries a flow of water of $4.0 \times 10^{-2}$ m$^3$ per second. The hose goes 10.0 m up a ladder to a nozzle which has an inside diameter of 3.0 cm. The water emerges from the nozzle at atmospheric pressure.

a) What is the velocity of the water at ground level?
b) What is the velocity of the water at the nozzle?
c) What is the (gauge) pressure of the water at ground level? You can ignore the viscosity of water.




Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$r_1 = \frac{d_1}{2} = 3.2\times10^{-2}\,m \qquad r_2 = \frac{d_2}{2} = 1.5\times10^{-2}\,m$$

a) $F = v_1 A_1 = v_2 A_2$

$$v_1 = \frac{F}{A_1} = \frac{F}{\pi r_1^2} = \frac{4\times10^{-2}\,m^3/s}{\pi(3.2\times10^{-2}\,m)^2} = \boxed{12.4\ m/s}$$

b)

$$v_2 = \frac{F}{A_2} = \frac{F}{\pi r_2^2} = \frac{4\times10^{-2}\,m^3/s}{\pi(1.5\times10^{-2}\,m)^2} = \boxed{56.6\ m/s}$$

c) $h_1 = 0$ $\quad h_2 = 10m$ $\quad P_2 = 1\,atm$ (absolute) $\underline{or}$ $= 0$ (gauge)

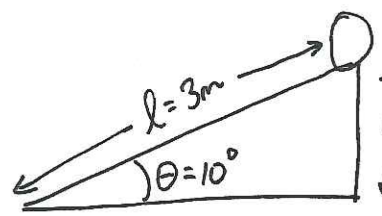
$$\therefore \quad P_1 + \frac{1}{2}\rho v_1^2 + 0 = \cancel{P_2}^{\,0} + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_1 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g h_2$$

$$= \frac{1}{2}\left(1000\,\frac{kg}{m^3}\right)\left(\left(56.6\,\frac{m}{s}\right)^2 - \left(12.4\,\frac{m}{s}\right)^2\right) + \left(1000\,\frac{kg}{m^3}\right)\left(9.8\,\frac{m}{s^2}\right)(10\,m)$$

$$= \boxed{1.62\times10^6\ Pa} \quad \text{(gauge pressure)}$$

## Problem 3.




A spherical ball of radius = 9 cm and mass 0.20 kg rolls without slipping down a slope with an incline of 10° and length 3 meters. It starts from rest. Note: the moment of inertia of a sphere about its center is $\frac{2}{5}MR^2$.

a) What is its velocity at the bottom of the incline?

b) What is its angular momentum at the bottom of the incline?

c) Since the angular momentum of the ball has increased, there must have been a net external torque on it. What force caused this torque?

d) If the sphere had slipped rather than rolled down the incline, would get it get to the bottom slower, faster, or in the same time? Explain.

$$E_i = E_f$$

a) $$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \qquad h = l\sin\theta, \quad I = \frac{2}{5}mR^2$$

rolling $\therefore$ $v = R\omega$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = mgl\sin\theta$$

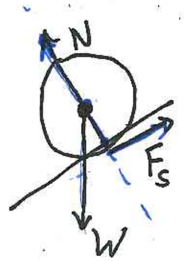
$$\frac{v^2}{2} + \frac{v^2}{5} = gl\sin\theta \qquad \therefore \quad v = \sqrt{\frac{10}{7}gl\sin\theta}$$

$$= \sqrt{\frac{10}{7}(9.8\,m/s^2)(3m)\sin 10^\circ} = \boxed{2.70\ m/s}$$

b) $$L = I\omega = \left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right) = \frac{2}{5}mRv = \frac{2}{5}(0.20\,Kg)(0.09m)(2.70\,m/s)$$

$$= \boxed{0.0194\ Kg\,m^2/s}$$

c) There are three forces on sphere: weight, normal force, friction.




← N and W exert no torque about center!

$\therefore$ must have been $F_s = \boxed{\text{Friction}}$ to cause torque

d) if slips not rolling: none of PE initial goes to $KE_{rot.}$ ($\frac{1}{2}I\omega$

$\therefore$ all goes to $\frac{1}{2}mv^2$ $\therefore$ $v$ is larger than with rolling

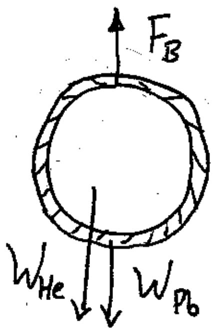
$$\left[v_{slip} = \sqrt{2gl\sin\theta} = 3.20\,m/s\right] \quad \therefore \quad \boxed{\text{Faster}}$$

# Problem 4.

An eccentric inventor decides to make a spherical helium-filled balloon out of lead (instead of something mundane like rubber). Assuming the balloon has a radius of 50 meters, how thin must the walls of the balloon be in order for it to *just* be able to float in air?

(perhaps this would be known as a 'Lead Zeppelin' ... sorry, I couldn't resist)

Useful information: the density of lead is $11.3 \times 10^3$ kg/m$^3$, the density of helium is 0.18 kg/m$^3$, and the density of air is 1.29 kg/m$^3$. The volume of material in a spherical shell of radius $R$ is $4\pi R^2 t$, where $t$ is the wall thickness. The volume enclosed by such a shell is $\frac{4}{3}\pi R^3$.




Archimedes' Principle : $F_B = W_{disp.}$

$$F_{net} = 0 \quad \therefore \quad F_B - W_{He} - W_{Pb} = 0$$

$$F_B = \rho_{air} V g \qquad W_{He} = \rho_{He} V g \qquad W_{Pb} = \rho_{Pb} V_{Pb}\, g$$

$$\therefore \quad \rho_{air} V g - \rho_{He} V g - \rho_{Pb} V_{Pb}\, g = 0$$

$$V = \frac{4}{3}\pi R^3$$

$$V_{Pb} = 4\pi R^2 t$$

$$\therefore \quad \frac{4}{3}\pi R^3 g(\rho_{air} - \rho_{He}) - \rho_{Pb} 4\pi R^2 t g = 0$$

$$t = \frac{R(\rho_{air} - \rho_{He})}{3\rho_{Pb}}$$

$$= \frac{(50\,m)\left(1.29\,\frac{Kg}{m^3} - 0.18\,\frac{Kg}{m^3}\right)}{3\,(11.3\ Kg/m^3)\times 10^3}$$

$$= 1.64 \times 10^{-3}\,m$$

$$= \boxed{1.64\,mm}$$

thin, but not absurdly so...