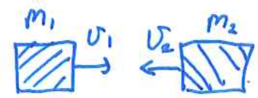
Problem 1.

A 5 kg block, moving to the right with a speed of 4 m/s on a frictionless table collides with a 15 kg block moving to the left with a velocity of 2 m/s. They collide and stick together.

a) What is the final velocity of the blocks?
b) How much kinetic energy is lost in the collision?
c) Energy is supposed to be conserved. Where did the missing kenetic energy go?
d) Assume that the collison takes place in 0.5 millisecond. What is the average net force (magnitude and direction) on the 5 kg block during the collision?
e) What is the average net force (magnitude and direction) on the 15 kg block during the collision?



a) Conservation of Momentum in collision; 1D collision
Define $x = +$ to right

$$\vec{P}_i = \vec{P}_f \rightsquigarrow m_1 v_1 - m_2 v_2 = (m_1 + m_2) v$$

$$v = \frac{m_1 v_1 - m_2 v_2}{(m_1+m_2)} = \frac{(5\,\text{Kg})(4\,\text{m/s}) - (15\,\text{Kg})(2\,\text{m/s})}{(5\,\text{Kg} + 15\,\text{Kg})} = \frac{(20-30)\,\text{Kg m/s}}{(20\,\text{Kg})}$$

$$\boxed{= -0.5\,\text{m/s} \quad (\therefore \text{to left})}$$

b) $$KE_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(5\,\text{Kg})(4\,\text{m/s})^2 + \frac{1}{2}(15\,\text{Kg})(2\,\text{m/s})^2$$
$$= 40\,\text{J} + 30\,\text{J} = 70\,\text{J}$$

$$KE_f = \frac{1}{2}(m_1+m_2)v^2$$
$$= \frac{1}{2}(20\,\text{Kg})(-0.5\,\text{m/s})^2 = 2.5\,\text{J}$$

$$\therefore \Delta KE = KE_f - KE_i = \boxed{-67.5\,\text{J}}$$ 67.5 J lost.

c) Energy lost to heat, sound, possible deformation of blocks

d) $$\vec{I} = \Delta\vec{p} = \vec{F}_{avg}\Delta t$$ (impulse)

$$F_{avg} = \frac{\Delta p}{\Delta t} = \frac{m_1 v - m_1 v_1}{\Delta t} = \frac{(5\,\text{Kg})(-0.5\,\text{m/s}) - (5\,\text{Kg})(4\,\text{m/s})}{0.5 \times 10^{-3}\,\text{s}}$$

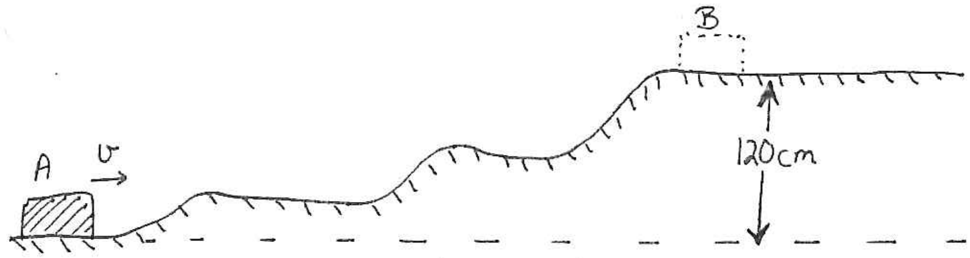
$$= \frac{-22.5\,\text{Kg m/s}}{5 \times 10^{-4}\,\text{s}} = \boxed{-4.5 \times 10^{4}\,\text{N}}$$

$\boxed{\text{to the left}}$

e) Newton's 3rd law : $\boxed{+4.5 \times 10^{4}\,\text{N} \text{ to the right}}$

or could calculate impulse on mass 2...

Problem 2.



A sled starts at the bottom of a 1.2 m high icy hill (point "A") with an initial speed of $v = 5$ m/s to the right, as shown. There is no friction between the sled and the hill, and you can neglect air resistance. Please use $g = 10$ m/s^2 here. Mass of sled = 2 Kg

a) What is the speed of the sled when it reaches the top of the hill (point "B")?
b) Starting at point B, the ice has been cleared away, and the ground is rough, so there is friction between the sled and the ground. The sled continues for 0.5 m farther before it comes to rest. What is the coefficient of kinetic friction between the sled and the ground?
c) In going from A to B, what is the net work done on the sled?
d) In going from A to B, what is the work done on the sled by the normal force?
e) In going from A to B, is the work done by gravity on the sled
i) positive, ii) negative, or iii) zero?

a) Conservation of Energy : $KE_i + PE_i = KE_f + PE_f$

define PE = 0 @ location A $\therefore \frac{1}{2}\cancel{m}v_A^2 + 0 = \frac{1}{2}\cancel{m}v_B^2 + \cancel{m}gh \quad \therefore v_A^2 = v_B^2 + 2gh$

$$v_B^2 = v_A^2 - 2gh$$
$$= (5\,m/s)^2 - 2(10\,\tfrac{m}{s^2})(1.2\,m)$$
$$= 25\,m/s^2 - 24\,m/s^2 = 1\,m/s^2$$
$$\boxed{v_B = 1\,m/s}$$

b) $W_{net} = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_B^2$

$F_f d\cos\theta = 0 - \frac{1}{2}mv_B^2$ } friction only force that does work

$-F_f d = -\frac{1}{2}mv_B^2$

$-\cancel{m}g\mu_k d = -\frac{1}{2}\cancel{m}v_B^2$

$$\mu_k = \frac{v_B^2}{2gd} = \frac{(1\,m/s)^2}{2(10\,\frac{m}{s^2})(0.5\,m)} = \frac{1}{10} = \boxed{0.1}$$

c) $W_{net} = \Delta KE = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$

$$= \frac{1}{2}m(v_B^2 - v_A^2) = \frac{1}{2}(2\,Kg)\left[(1\,m/s)^2 - (5\,m/s)^2\right] = \boxed{-24\,J}$$

d) N is $\perp$ to displacement everywhere on path $\therefore \boxed{W_N = 0}$

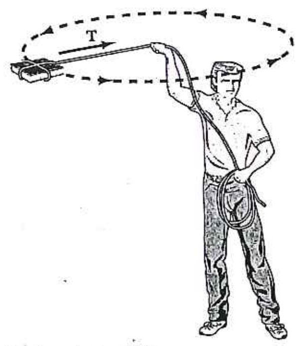
e) only other force acting between A and B is gravity $\therefore W_g = W_{net} = -24\,J$

$\therefore$ $\boxed{\text{negative}}$

or $W_g = -\Delta PE = -mgh$

$$= -(2\,Kg)(10\,\tfrac{m}{s^2})(1.2\,m) = -24\,J$$

Problem 3.



Professor Armstrong ties a book with a string and swings it around his head in a horizontal circle (see figure) at a constant rate. The length of the string of the yo-yo is 1.5 m, and the book has a mass of 400 grams.
For the moment, ignore gravity.

a) If the book makes a complete revolution in 0.5 seconds, what is the speed of the book?
b) What is the acceleration of the book? Please give both magnitude and direction.
c) What is the tension in the string?
d) How much work does the tension in the string do on the book?
e) He now decides to swing the same book in a vertical circle. Now you must take gravity (the weight of the book) into consideration. Draw a free body diagram of the book when it is at the top of its circular path.
f) Assume that at this instant it is moving with the same speed as you calculated in part a). What is the tension in the string now?

a) $v = R\omega = R(2\pi f) = \frac{R2\pi}{T} = \frac{(1.5m)2\pi}{0.5s} = \boxed{6\pi\ m/s} = 18.8\ m/s$

b) centripetal acceleration $a_c = \frac{v^2}{R} = \frac{(6\pi\, m/s)^2}{1.5m} = 24\pi^2\ m/s^2 = \boxed{237\ m/s^2}$

direction : $\boxed{\text{towards center of circle}}$

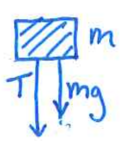
c) 

FBD

$\therefore\ \vec{F}_{net} = m\vec{a}$

$T = ma_c = (0.4\,Kg)(24\pi^2 m/s^2) = 9.6\pi^2 N = \boxed{94.7N}$

d) T is perpendicular to displacement $(\cos\theta = 0)$ $\therefore \boxed{W = 0}$ tension does no work. $\Delta KE = 0$ as $W_{net} = 0$

e) FBD: 

f) $\vec{F}_{net} = m\vec{a}$

$\therefore\ T + mg = \frac{mv^2}{R}$

$T = \frac{mv^2}{R} - mg = m\left(\frac{v^2}{R} - g\right)$

$= (0.4\,Kg)\left(237\frac{m}{s^2} - 9.8\,m/s^2\right)$

$= \boxed{90.9\,N}$

less than vertical circle, and also less than @ bottom of circle (which would have max. tension)