Problem 1.

A 1.5 kg textbook slides along the horizontal surface of a table. The initial speed of the textbook is 2.0 m/s. It slides for a distance of 0.8 m, until it comes to rest (there is friction between the book and the table).

a) What is the initial kinetic energy of the book?

b) What is the total (net) work done on the book?

c) What is the work done by gravity?

d) What is the work done by friction?

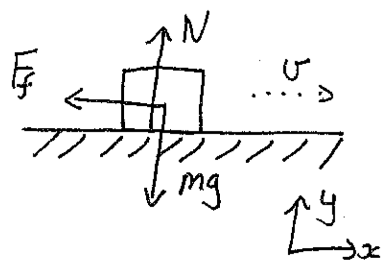
e) What is the coefficient of kinetic friction between the book and the table?

a) $KE_i = \frac{1}{2} m v^2 = \frac{1}{2}(1.5\,\text{kg})(2.0\,\text{m/s})^2 = \boxed{3.0\,\text{J}}$

b) $KE_f = 0 \quad \therefore \quad W_{net} = \Delta KE = KE_f - KE_i = 0 - 3\,\text{J} = \boxed{-3.0\,\text{J}}$

c) $W_{gravity} = (mg) d \cos 90^\circ = 0$

d)



neither N nor mg do any work ($\theta = 90^\circ$) $\therefore W_{net} = W_{friction}$

$W_{friction} = \boxed{-3.0\,\text{J}}$

e) $\sum \vec{F} = m\vec{a} \rightarrow y: \; N - mg = 0$

$\therefore N = mg$

$F_f = \mu_K N = \mu_K mg$

$W_{friction} = F_f \cdot d \cdot \cos(180^\circ) = -\mu_K m g d$

$\therefore \mu_K = -\frac{W_{friction}}{mgd} = -\frac{(-3\,\text{J})}{(1.5\,\text{kg})(9.8\,\frac{\text{m}}{\text{s}^2})(0.8\,\text{m})}$

$= \boxed{0.255}$

Problem 2.

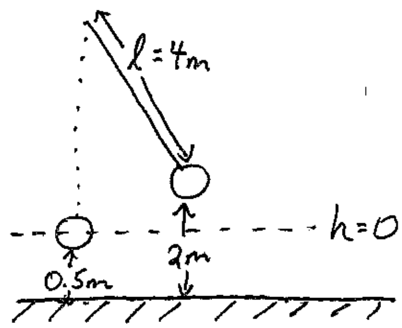
In the "nose-basher" demonstration, Professor Armstrong used a pendulum consisting of an iron ball which is attached via a cable to the ceiling. The ball starts off at rest, and is initially at a height of 2.0 m above the ground. At the bottom of its swing the ball is at a height of 0.5 m above the ground. The cable is 4.0 m long.

a) What is the speed of the iron ball at the bottom of its swing?

b) What is the angular velocity of the ball at this instant?

c) What is the centripetal acceleration of the ball at this instant? Give the magnitude and direction.

a) Pendulum: energy conserved (ignoring any air resistance)



$KE_i + PE_i = KE_f + PE_f$

$KE_i = 0$

choose $h=0$ @ bottom of swing

$\therefore PE_i = mgh \quad h = 2.0 - 0.5 = 1.5\,m$

$$KE_f = \frac{1}{2}mv_f^2 = mgh \quad \therefore v_f = \sqrt{2gh} = \sqrt{2(9.8\tfrac{m}{s^2})(1.5m)}$$

$$= \boxed{5.42\ m/s}$$

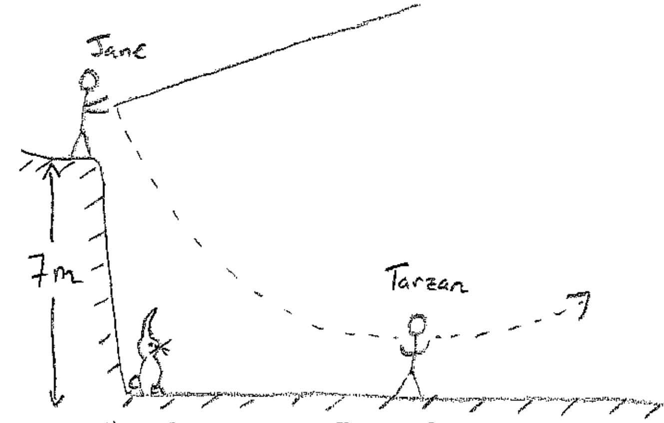
b) $v = R\omega \qquad \omega = \frac{v}{R} = \frac{5.42\ m/s}{4.0m} = \boxed{1.36\ rad/s}$

c) $a_c = \frac{v^2}{R} = \frac{(5.42\ m/s)^2}{4.0m} = \boxed{7.35\ m/s^2}$

Direction = $\boxed{\text{upwards}}$

(centripetal = "center-pointing")

Problem 3.



Tarzan is threatened by a killer bunny rabbit. He is standing still, petrified with fear. Jane, who is standing on a ledge 7 m above Tarzan, grabs a conveniently located vine, and swings down to rescue him. She grabs hold of him at the bottom of her swing (her velocity at this point is entirely horizontal). They swing off together, out of harm's way. Tarzan's mass is 70 kg, and Jane is a svelte 50 kg.

a) What is Jane's speed just before she collides with Tarzan?

b) What is their speed just after they collide?

c) How high do they swing up together?

d) How much mechanical energy was lost in their collision?

(choose PE = 0 @ ground)

a) energy conserved: $KE_i + PE_i = KE_f + PE_f$

(pendulum!) $0 + m_J g h = \frac{1}{2} m_J v_J^2 + 0$

$\therefore v_J = \sqrt{2gh} = \sqrt{2(9.8\,m/s^2)(7\,m)} = \boxed{11.7\,m/s}$

b) Inelastic collision (stick together!)

$\therefore$ mechanical energy not conserved

momentum is conserved (no net external forces)

all momentum in horizontal direction

$\vec{p}_i = \vec{p}_f \qquad m_J v_J = (m_J + m_T) v$

$v = \frac{m_J v_J}{(m_J + m_T)} = \frac{(50\,kg)(11.7\,m/s)}{(50\,kg + 70\,kg)} = \boxed{4.88\,\frac{m}{s}}$

c) new pendulum, of mass $(m_J + m_T)$

$KE_i + PE_i = KE_f + PE_f$

$\frac{1}{2}(m_J + m_T) v^2 = 0 + (m_J + m_T) g h'$

$h' = \frac{v^2}{2g} = \frac{(4.88\,m/s)^2}{2(9.8\,m/s^2)} = \boxed{1.21\,m}$

d) $\Delta KE = \frac{1}{2} m_J v_J^2 - \frac{1}{2}(m_J + m_T) v^2$

$= 3.42 \times 10^3\,J - 1.43 \times 10^3\,J = \boxed{1.99 \times 10^3\,J}$

Problem 4.

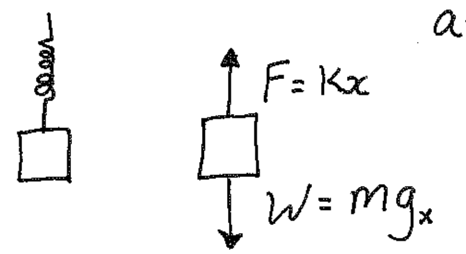
Captain Jean-Luc Picard is standing on the surface of the previously unknown planet Xantac. He knows that Xantac has a radius of 2000 km, but he does not know its mass. To determine the planet's mass, he suspends a 10 kg mass vertically using a spring with a spring constant of 300 N/m, and he finds that the spring stretches by 10 cm from its unstretched length.

a) What is the acceleration due to gravity at Xantac's surface?

b) What is Xantac's mass?

c) Picard observes that Xantac has a moon which orbits the planet once every 12 hours. How far away is that moon?

a)



$a = 0 \quad \therefore \quad Kx - mg_x = 0$

$$g_x = \frac{Kx}{m} = \frac{(300\,\frac{N}{m})(0.1\,m)}{10\,Kg} = \boxed{3.0\,m/s^2}$$

b) $W = mg_x = \dfrac{GM_x m}{R_x^2} \quad \therefore \quad g_x = \dfrac{M_x G}{R_x^2}$

weight

$$M_x = \frac{g_x R_x^2}{G} = \frac{(3\,m/s^2)(2000\times10^3\,m)^2}{\left(6.67\times10^{-11}\,\frac{Nm^2}{Kg^2}\right)} = \boxed{1.80\times10^{23}\,Kg}$$

c) Kepler's 3rd law:

$$T^2 = \frac{4\pi^2 r^3}{GM_x}$$

(amazing what one learns @ Star Fleet Academy)

$$r = \left[\frac{GM_x T^2}{4\pi^2}\right]^{1/3}$$

$$= \left[\frac{\left(6.67\times10^{-11}\,\frac{Nm^2}{Kg^2}\right)(1.80\times10^{23}\,Kg)\left(12\,hr\times\frac{3600\,s}{hr}\right)^2}{4\pi^2}\right]^{1/3}$$

$$= \boxed{8.28\times10^6\,m} = 8280\,Km$$