Problem 1.

A car starts from rest and travels in a straight line for 12 seconds with a uniform acceleration of 1.5 m/s^2. The driver then applies the brakes, causing the car to slow with a deceleration of 2.0 m/s^2, until it comes to a full stop.

a) What is the maximum speed of the car?
b) How far is the car away from its initial location when it stops?
c) What is the total elapsed time?

a) $v_0 = 0$ $v = v_0 + at = 0 + (1.5\,m/s^2)(12s) = \boxed{18\,m/s}$

b) while accelerating ($x_0 = 0$) : $x_1 = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2 = \frac{1}{2}(1.5\,m/s^2)(12s)^2 = 108m$

while decelerating : here $v_0 = 18\,m/s$, $v = 0$ for full stop, $a = -2.0\,m/s^2$

$\therefore\ v^2 = v_0^2 + 2a\Delta x$

$\therefore\ \Delta x = \dfrac{v^2 - v_0^2}{2a} = \dfrac{0^2 - (18\,m/s)^2}{2(-2.0\,m/s^2)} = 81\,m = x_2$

total distance $= x_1 + x_2$
$= \boxed{189m}$

c) while accelerating : $t_1 = 12s$ (given)

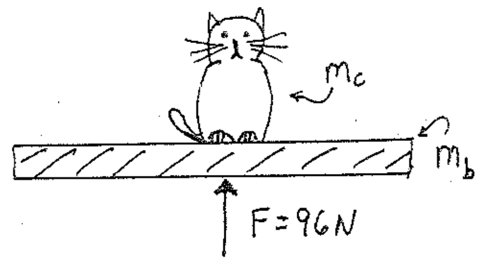
while decelerating : $v = v_0 + at_2$ $\left\{ \begin{array}{l} v_0 = 18\,m/s \\ v = 0 \end{array} \right.$

$\therefore\ t_2 = \dfrac{v - v_0}{a}$

$= \dfrac{0 - 18\,m/s}{-2.0\,m/s^2} = 9s$

total time $= 12s + 9s = \boxed{21s}$

Problem 2.



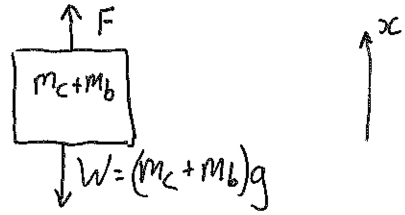
A plump cat, mass $m_c = 6$ kg sits on top of a board, which has a mass of $m_b = 2$ kg. You are pushing up on the board from below with a force of $F = 96$ N. You can use the approximate value $g = 10$ m/s^2.

a) What is the acceleration of the board?

b) What is the force (magnitude and direction) that the board exerts on the cat?

c) What is the force (magnitude and direction) that the cat exerts on the board?

a) cat & board as system FBD:



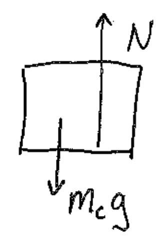
$\vec{F}_{net} = m\vec{a}$

$\hookrightarrow \quad F - W = (m_c + m_b)a$

$$a = \frac{F - (m_c+m_b)g}{(m_c+m_b)} = \frac{96N - (2+6\text{ Kg})(10\frac{m}{s^2})}{(2\text{Kg} + 6\text{ Kg})} = \boxed{+2 m/s^2 \text{ (upwards)}}$$

b) now, consider cat as system:

FBD :



$\vec{F}_{net} = m\vec{a}$

a is same is above (cat stays on board)

$\therefore \quad N - m_c g = m_c a$

$N = m_c(a+g)$

$= 6\text{Kg}(2 m/s^2 + 10 m/s^2)$

$= +72N$

$\boxed{\text{board exerts 72N on cat, upwards direction}}$

c) Sir Isaac's 3rd Law: $\vec{F}_{bc} = -\vec{F}_{cb}$

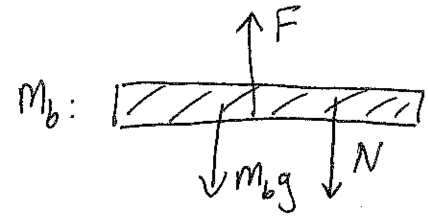
force board exerts on cat

force cat exerts on board

$\boxed{\text{Cat exerts 72N in downwards direction on board}}$

c) <u>alternate solution</u> (if you forgot Newton's 3rd law)

FBD of board:



F = person's force
$m_b g$ = weight of board
N = normal force cat exerts on board.

$$F_{net} = m_b a$$

$$F - m_b g - N = m_b a$$

$$\therefore \; N = F - m_b g - m_b a$$

$$= 96N - (2kg)(10\,m/s^2) - (2\,kg)(2\,m/s^2)$$

$$= 96N - 20N - 4N = \boxed{72N}$$

(downward, as drawn in FBD)

but clearly easier to just apply the 3rd law & get the result in one step!

Problem 3.

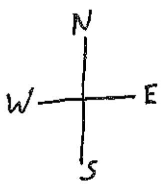
Long-distance swimmer Diana Nyad decides to swim across the James River. She swims at a speed of 0.50 m/s with respect to the water. The current in the river is 0.30 m/s and flows from West to East. She starts on the South bank of the river and wishes to arrive at the opposite shore at a spot directly North of her starting point.

a) In what direction (with respect to the water) must she swim?

b) The river is 2.4 kilometers wide at her location. How many minutes will it take her to reach the other side?

a) Relative velocity problem. Define: $\vec{v}_{wg}$ = water with respect to ground = current

$= 0.30\ m/s$ East



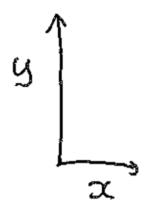
$\therefore \vec{v}_{Dg} = \vec{v}_{Dw} + \vec{v}_{wg}$

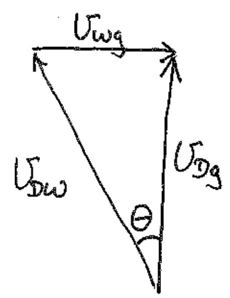
$\vec{v}_{Dg}$ = Diana with respect to ground

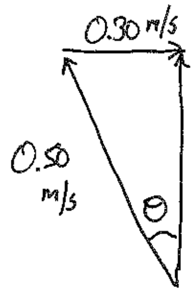
Direction = North

$\vec{v}_{Dw}$ = Diana with respect to water

$= 0.50\ m/s$, Direction unknown







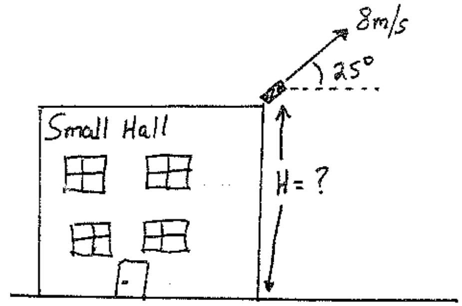
$\therefore \sin\theta = \dfrac{0.30\ m/s}{0.50\ m/s} \qquad \theta = \sin^{-1}\left(\dfrac{0.3}{0.5}\right) = 36.9^\circ$

she must swim $\boxed{36.9^\circ \text{ West of North}}$

b) $v_{Dg} = v_{Dw}\cos\theta = \left(0.5\frac{m}{s}\right)\cos(36.9^\circ) = 0.4\ m/s$

$$t = \frac{\Delta x}{v} = \frac{2400\,m}{0.4\ m/s} = 6000\,s \times \frac{1\,min}{60\,s} = \boxed{100\ min}$$

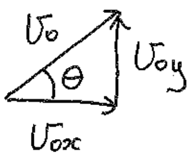
Problem 4.



A physics student throws a brick off the top of a building. The brick is thrown at an angle of 25° to the horizontal with an initial speed of 8 m/s.

a) Find the horizontal and vertical components of the initial velocity.
b) The brick takes 2 seconds to hit the ground. How tall is the building?
c) What is the velocity of the brick (magnitude and direction) in the instant before it hits the ground?

a)



$$v_{0x} = v_0 \cos\theta = (8\,m/s)\cos 25° = \boxed{7.25\,m/s}$$

$$v_{0y} = v_0 \sin\theta = (8\,m/s)\sin 25° = \boxed{3.38\,m/s}$$

b) Projectile motion $\quad y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

choose: y up, x right, origin @ base of building $\quad \therefore \; y_0 = H,\ y = 0,\ a_y = -g,\ t = 2s$

$$0 = H + v_0 \sin\theta\, t - \frac{1}{2}gt^2$$

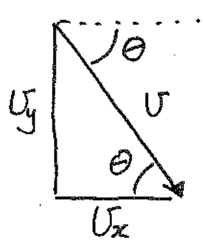
$$\therefore H = \frac{1}{2}gt^2 - v_0 \sin\theta\, t = \frac{1}{2}(9.8\,m/s^2)(2s)^2 - (3.38\,\tfrac{m}{s})(2s)$$

$$= \boxed{12.8\,m}$$

c) $v_x = v_{0x} = 7.25\,m/s$

$$v_y = v_{0y} - gt = 3.38\,m/s - (9.8\,m/s^2)(2s) = -16.2\,m/s$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(7.25\,m/s)^2 + (-16.2\,m/s)^2} = \boxed{17.75\,m/s}$$



$$\theta = \tan^{-1}\left(\frac{|v_y|}{|v_x|}\right) = \boxed{65.9° \text{ below horizontal}}$$