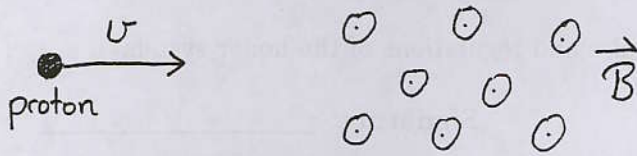


## Problem 1.

A proton is moving to the right with speed  $v$ . It enters a uniform magnetic field  $\vec{B}$  which points out of the page. The magnitude of  $B$  is  $0.2\text{ T}$ .



a) What is the direction of the force on the proton? (into page, out of page, up, down, to the left, or to the right)?

b) Now assume there is also a uniform electric field present in the same region. In what direction must  $\vec{E}$  point if the proton is to proceed undeflected?

c) It is found that an electric field of strength  $1000\text{ V/m}$  is needed in order that the proton not be deflected. What is the proton's speed  $v$ ?

a)  $\vec{F} = q\vec{v} \times \vec{B}$  use Right Hand Rule:  $\vec{v} \times \vec{B} = \text{down the page}$   
 proton has positive charge  $\therefore \vec{F} = \boxed{\text{down the page}}$

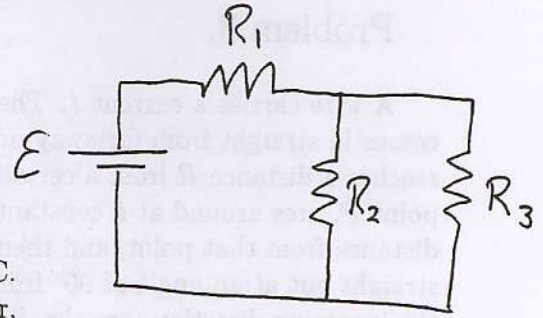
b) undeflected  $\therefore \Sigma \vec{F} = 0 \quad \therefore \vec{F}_E = -\vec{F}_B$   
 $\therefore q\vec{E} = \text{up the page} \quad \therefore \vec{E} = \boxed{\text{up the page}}$

c)  $\vec{F}_E = -\vec{F}_B$   
 $F_E = F_B \quad qE = qvB \sin \theta = qvB$   
 $\therefore v = E/B = \frac{1000\text{ V/m}}{0.2\text{ T}} = \boxed{5000\text{ m/s}}$

(check of units:  $T = \frac{N}{A \cdot m} \quad \therefore \frac{V/m}{T} = \frac{V \cdot A \cdot m}{N \cdot m} = \frac{VA}{N} = \frac{W}{N} = \frac{Nm/s}{N} = m/s$ )

## Problem 2.

Consider the circuit shown, with  $\mathcal{E} = 10 \text{ V}$ ,  
 $R_1 = 100 \Omega$ ,  $R_2 = 200 \Omega$  and  $R_3 = 200 \Omega$ .



- What is the current passing through  $R_1$ ?
- How much heat is generated in  $R_1$  in 10 seconds?
- If the battery were replaced by a 110 V, 60 Hz A.C. power source, and  $R_1$  was replaced by a 122 mH inductor, what would be the peak current in the inductor?

a)  $R_2$  &  $R_3$  are in parallel  $\therefore \frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{200\Omega} + \frac{1}{200\Omega} \therefore R_p = 100\Omega$

$R_1$  is in series with the  $R_2, R_3$  combination

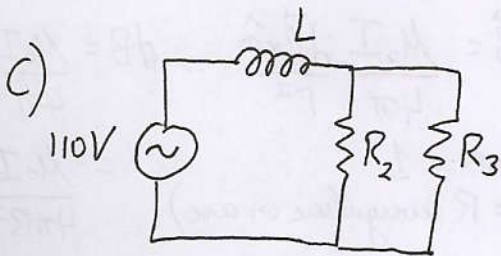
$$\therefore R_{eq} = R_1 + R_p = 100\Omega + 100\Omega = 200\Omega$$

$$\mathcal{E} = I R_{eq} \therefore I = \frac{\mathcal{E}}{R_{eq}} = \frac{10\text{V}}{200\Omega} = 0.05\text{A} = \boxed{50\text{mA}} = I_1$$

( $I_{\text{Battery}} = I_1$ )

b)  $P_1 = I_1^2 R_1 = (5 \times 10^{-2}\text{A})^2 (100\Omega) = 0.25\text{W}$

$$\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{E}{t} \therefore E = Pt = (0.25\text{W})(10\text{s}) = \boxed{2.5\text{J}}$$



$$X_L = \omega L = 2\pi f L = (2\pi)(60\text{Hz})(122 \times 10^{-3}\text{H}) = 46\Omega$$

$R_2, R_3$  in parallel - replace by  $R_p = 100\Omega$  as above

Total impedance:  $Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R_p^2 + X_L^2}$  (as  $X_C = 0$  since no capacitor)

$$= ((100\Omega)^2 + (46\Omega)^2)^{1/2} = 110\Omega$$

$$\mathcal{E}_0 = I_0 Z$$

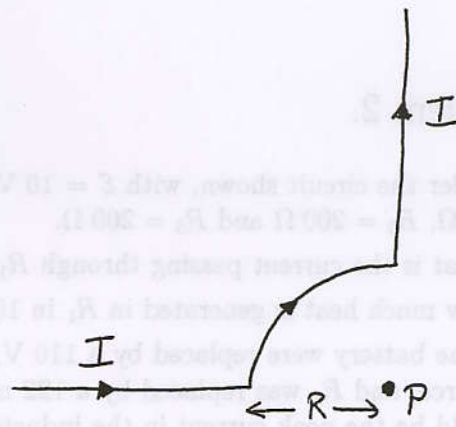
$$\therefore \mathcal{E}_{\text{rms}} = I_{\text{rms}} Z$$

$$I_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{110\text{V}}{110\Omega} = 1\text{A}$$

$$I_0 = \sqrt{2} I_{\text{rms}} = \sqrt{2}\text{A} = \boxed{1.41\text{A}}$$

### Problem 3.

A wire carries a current  $I$ . The wire comes in straight from far away until it reaches a distance  $R$  from a certain point  $P$ , arcs around at a constant distance from that point, and then exits straight out at an angle of  $90^\circ$  from the incoming direction (see the diagram). Consider the magnetic field at the point  $P$ .

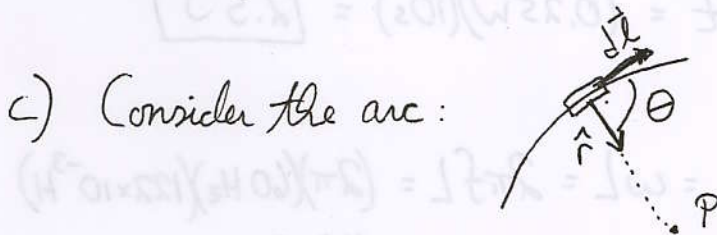


- What is the contribution to the magnetic field at  $P$  from the long straight sections? If you do no calculations, briefly explain why.
- What is the direction of the magnetic field at  $P$ ?
- Use the Biot-Savart law to calculate the magnitude of the magnetic field at  $P$ , if  $R = 50 \text{ cm}$  and  $I = 5 \text{ A}$ ?

a) Biot-Savart law: 
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

for long straight sections  $d\vec{l}$  is parallel to  $\hat{r}$   $\therefore d\vec{l} \times \hat{r} = 0 \therefore d\vec{B} = 0$   
 ↙ current element      ↘ unit vector from current element to point P.

b) use Right Hand Rule:  $\vec{B}$  at  $P$  is into the page



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} \therefore dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin\theta}{R^2}$$

( $\sin\theta = 1$   
 $r = R$  everywhere on arc)  $= \frac{\mu_0 I}{4\pi R^2} dl$

$$B = \int_0^{\pi/2} dB \quad (\text{circumference of arc} = \frac{2\pi R}{4} = \frac{\pi R}{2})$$

$$= \frac{\mu_0 I}{4\pi R^2} \int_0^{\pi/2} dl = \frac{\mu_0 I l}{4\pi R^2} \Big|_0^{\pi/2} = \frac{\mu_0 I}{4\pi R^2} \frac{\pi R}{2} = \frac{\mu_0 I}{8R}$$

Note: result is  $1/4$  of  $B$  at center of a full loop, as one would expect...

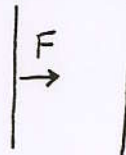
$$= \frac{(4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}})(5 \text{ A})}{8(0.50 \text{ m})}$$

$$= \frac{\pi}{2} \times 10^{-6} \text{ T} = \boxed{1.57 \times 10^{-6} \text{ T}}$$

## 4. Multiple Choice

a) Two long straight wires are parallel to each other, and each carries current. The wire on the left side experiences a force to the right. What can one say about the directions of the currents in the wires?

- i) They are the same
- ii) They are opposite each other
- iii) Not enough information is provided



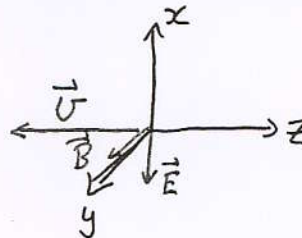
*(opposite currents attract ...  
check via Right Hand Rule..)*

b) At some instant, a radio wave has an electric field vector pointing in the negative  $x$  direction and a magnetic field vector pointing in the positive  $y$  direction. In what direction is the wave travelling?

- i) Positive  $x$
- ii) Negative  $x$
- iii) Positive  $y$
- iv) Negative  $y$
- v) Positive  $z$
- vi) Negative  $z$
- vii) Radio waves have no direction

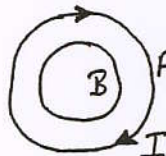
$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

*$\vec{v}$  in direction of  $\vec{S}$*



c) Two coils of wire are located concentrically in the same plane; coil A is larger than coil B. There is a clockwise, decreasing current in coil A. What is the direction of the current induced in coil B?

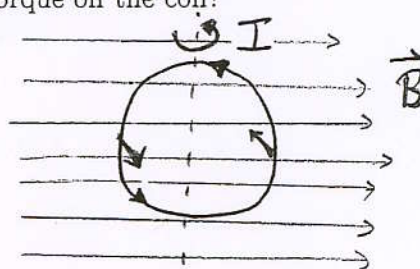
- i) Clockwise
- ii) Counterclockwise
- iii) There is no current



*A creates flux into page  
inside B  $\therefore$  Lenz's law says  
EMF in B to create more flux  
into page  $\therefore$  CW current  
induced*

d) A circular loop of wire is in the plane of the page, and carries a counterclockwise current. There is a magnetic field pointing to the right. What is the direction of the torque on the coil?

- i) To the left
- ii) To the right
- iii) Into the page
- iv) Out of the page
- v) Up the page
- vi) Down the page



$$\vec{\mu} = \text{out of page}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$= \text{up the page}$$

*(Right Hand Rule)*

e) A material is inserted into a solenoid which carries a current. The magnetic field is found to increase by a factor of 300 compared to that without the material. The material is most likely to be

- i) Dielectric
- ii) Paramagnetic
- iii) Diamagnetic
- iv) Ferromagnetic