Problem 1.

a) State one version of the Second Law of Thermodynamics.

b) A Carnot engine performs work at the rate of 580 W with an input of 1310 cal of heat per second. If the temperature of the heat source is 540°C, at what temperature is the waste heat exhausted?

c) If the engine cycle were run backwards in order to turn this into a heat pump, with the same operating temperatures and the same rate of work as above, how much heat is extracted from the cold reservoir in one hour?

a) $\Delta S_{closed} \geq 0$: the entropy can never decrease for a closed system or any of the 3 other versions discussed in lecture...

b) $T_H = 540°C = 813\,K$

$$\frac{Q_H}{t} = 1310\,\frac{cal}{s} \times \frac{4.186\,J}{cal} = 5.48\ KJ/s$$

$$\frac{W}{t} = 580\,\frac{J}{s}$$

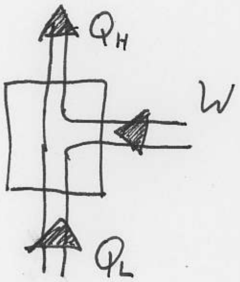
$$\varepsilon = \frac{W}{Q_h} = \frac{W/t}{Q_h/t} = \frac{580\,J/s}{5.48\,KJ/s} = 0.106$$

$$\varepsilon_{Carnot} = 1 - \frac{T_L}{T_H} = 0.106 \quad \therefore \quad T_L = T_H(1-0.106)$$

$$= (813K)(0.894) = \boxed{727\,K}$$

$$= 454\,°C$$

c) heat pump :



$$Q_L + W = Q_H$$

$$Q_L = Q_H - W \Rightarrow \frac{Q_L}{t} = \frac{Q_H}{t} - \frac{W}{t}$$

$$= 5.48\,\frac{KJ}{s} - \frac{580\,J}{s} = 4.90\,\frac{KJ}{s}$$

in one hour $\quad Q_L = \frac{Q_L}{t}\cdot(1\ hour)$

$$= \left(4.90\times10^3\,\frac{J}{s}\right)\left(3600\,\frac{s}{hr}\right)(1\,hr)$$

$$= \boxed{17.6\ MJ}$$

or 4220 Kcal

Problem 2.

A chunk of ice of mass 100 g at a temperature of 0°C is dropped into an insulated flask containing 0.5 kg of liquid water which is at some unknown initial temperature. Once thermal equilibrium is reached, you observe a mixture of ice and water, in which 70% of the original ice cube has melted.

Possibly useful data: specific heat of water = 4186 J/kg·C°, specific heat of ice = 2100 J/kg·C°, latent heat of fusion of water = 333 kJ/kg, latent heat of vaporization of water = 2260 kJ/kg.

a) Is this process reversible or irreversible?

b) What was the inital temperature of the water?

c) What is the change in entropy of the ice?

d) What is the change in entropy of the water?

a) $\boxed{\text{irreversible}}$ (there is heat flow due to ΔT)

b) Calorimetry. $\Sigma Q = 0$ $\quad m_{ice} f_{melt} L_f + m_w c_w (0° - T_i) = 0$

f_{melt} = fraction ice melted

$$\therefore T_i = \frac{m_{ice} f_{melt} L_f}{m_w c_w}$$

$$T_i = \frac{(0.1\,\text{Kg})(0.7)(333\times10^3\,\text{J/Kg})}{(0.5\,\text{Kg})(4186\,\frac{\text{J}}{\text{Kg}\cdot\text{C}°})} = \boxed{11.1\,°\text{C}} = 284\,\text{K}$$

c) $$\Delta S_{ice} = \int \frac{dQ_{ice}}{T_{ice}} = \frac{\Delta Q_{ice}}{T_{ice}} = \frac{(0.1\,\text{Kg})(0.7)(333\times10^3\,\frac{\text{J}}{\text{Kg}})}{273\,\text{K}} = \boxed{+85.4\,\frac{\text{J}}{\text{K}}}$$

d) $$\Delta S_w = \int \frac{dQ_w}{T_w} = \int \frac{m_w c_w\, dT_w}{T_w} = m_w c_w \int_{T_i}^{T_f} \frac{dT_w}{T_w} = m_w c_w \ln\left(\frac{T_f}{T_i}\right)$$

$$= (0.5\,\text{Kg})\left(4186\,\frac{\text{J}}{\text{Kg}\cdot\text{C}°}\right)\ln\left(\frac{273\,\text{K}}{284\,\text{K}}\right)$$

$$= \boxed{-82.7\,\frac{\text{J}}{\text{K}}}$$

or $$\Delta S_w \simeq \frac{m_w c_w \Delta T}{T_{average}}$$ (since ΔT is small)

$$= \frac{(0.5\,\text{Kg})\left(4186\,\frac{\text{J}}{\text{Kg}\cdot\text{C}°}\right)(-11.1\,\text{K})}{\left(\frac{284+273}{2}\,\text{K}\right)} = -83.4\,\frac{\text{J}}{\text{K}}$$

note : ΔS_{system} = negative, as should be for irreversible process

Problem 3.

Consider a sample of krypton, which is a monatomic gas, and can be considered to be ideal. It is initially at a pressure of 200 kPa, a temperature of 350 K, and is contained in a volume of 0.2 m^3.

a) How many molecules are in the sample?

b) The gas is expanded so as to double its volume. The gas is now at a pressure of 63 kPa. What is its new temperature?

c) If this expansion was done adiabatically, how much work was done by the gas?

d) The gas is now compressed isobarically back to its original volume. How much work is done *on* the gas in this step?

e) Does the internal energy *increase*, *decrease* or *stay the same* in this step?

f) Is heat *added to* or *extracted from* the gas in this step?

a) $PV = NKT \quad N = \frac{PV}{KT} = \frac{(200\times10^3 \frac{N}{m^2})(0.2m^3)}{(1.38\times10^{-23}\frac{J}{K})(350K)} = \boxed{8.28\times10^{24}}$

$= 13.7$ mol

b) $PV = nRT \quad T = \frac{PV}{nR} = \frac{(63\times10^3 \frac{N}{m^2})(0.4m^3)}{(13.7 mol)(8.314 \frac{J}{mol\cdot K})} = \boxed{221 K}$

c) Adiabatic $\therefore Q=0 \qquad Q = \Delta E_{int} + W \rightarrow W = -\Delta E_{int} \qquad E_{int} = \frac{3}{2}nRT$

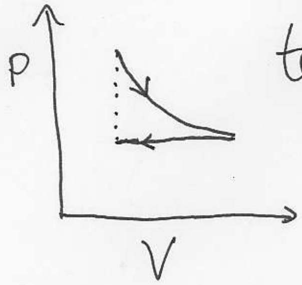
$\therefore W = -\left[\frac{3}{2}nRT_f - \frac{3}{2}nRT_i\right] = \frac{3}{2}(13.7 mol)(8.314 \frac{J}{mol\cdot K})(350K - 221K)$

$= \boxed{+22.0 KJ}$

d) $dW = PdV \quad P = \text{constant} \therefore W = P\Delta V$

$= (63\times10^3 \frac{N}{m^2})(0.2m^3 - 0.4m^3) = -12.6 KJ$

this is work done by the gas $\therefore$ work done on the gas $= \boxed{+12.6 KJ}$

e) 

temperature has decreased (think of where isotherms are on PV diagram)

$\therefore$ Eint has $\boxed{\text{decreased}}$

f) Since both ΔE_{int} & W done by gas are negative (see parts d, e)

& $Q = \Delta E_{int} + W$

$\therefore Q =$ negative $\therefore$ heat is $\boxed{\text{extracted}}$ from gas.

Problem 4.

Consider a gaseous mixture of nitrogen (N_2) and carbon dioxide (CO_2) at some unknown temperature. The molar mass of N_2 is 28 g and that of CO_2 is 44 g.

a) What is the ratio of the average kinetic energies of the nitrogen and carbon dioxide molecules in the gas?

b) What is the ratio of the root-mean-square speeds of the nitrogen and carbon dioxide molecules in the gas?

c) If the root-mean-square speed of the nitrogen molecules is measured to be 550 m/s, what is the temperature of the gas mixture?

a) $\overline{K} = \frac{3}{2} kT$ gases @ same temperature $\therefore$ same $\overline{K}$

ratio : $\boxed{1:1}$

b) $v_{rms} = \left(\frac{3kT}{m}\right)^{1/2}$ $\therefore$ $\dfrac{v_{rms}^{N_2}}{v_{rms}^{CO_2}} = \dfrac{\left(\frac{3kT}{m_{N_2}}\right)^{1/2}}{\left(\frac{3kT}{m_{CO_2}}\right)^{1/2}} = \left(\frac{m_{CO_2}}{m_{N_2}}\right)^{1/2}$

$= \left(\frac{44 \text{ g/mole}}{28 \text{ g/mole}}\right)^{1/2}$

Nitrogen molecules are 25 % faster than CO_2 molecules @ same T. $\Longrightarrow$ $= \boxed{1.25}$

c) $v_{rms} = \left(\frac{3kT}{m}\right)^{1/2}$

$v_{rms}^2 = \frac{3kT}{m}$ $\therefore$ $T = \frac{m v_{rms}^2}{3k}$

$m = \frac{28 \text{ g}}{\text{mole}} \Rightarrow \frac{28 \times 10^{-3} \text{ Kg}}{6.022 \times 10^{23} \text{ molecules}} = 4.65 \times 10^{-26} \text{ Kg}$

$T = \dfrac{(4.65 \times 10^{-26} \text{ Kg})(550 \frac{m}{s})^2}{3(1.38 \times 10^{-23} \frac{J}{K})} = \boxed{340 \text{ K}}$

or $67\,^{\circ}C$