Problem 1.

An ideal monoatomic gas at 10 °C is maintained in a constant volume of 10 litres (10^{-2}m^3) while its pressure is increased from 1 atm (10^5 Pa) to 2 atm (2×10^5 Pa).

a) Calculate the new temperature, the work done in this step, and the heat flow. Is heat added to or removed from the gas?

b) The gas is then allowed to expand to a new volume of 30 litres, in such a way as to stay at this new temperature. What is the total work done in this step? Is it done *on* the gas or *by* the gas?

c) Calculate the heat flow in this second step, and indicate if it is added to or removed from the gas.

d) Sketch the entire process on a $P - V$ diagram.

a) $PV = nRT$, $n = \text{constant}$ $\therefore$ $\dfrac{P_A V_A}{T_A} = \dfrac{P_B V_B}{T_B}$ $\therefore$ $T_B = T_A\left(\dfrac{P_B}{P_A}\right)$

$T_A = 10°C = 283\text{ K}$

$= 283\text{K}\left(\dfrac{2\times10^5}{10^5}\right) = \boxed{566\text{ K}}$

$W = \int P dV = \boxed{0}$ as $V = \text{constant}$

$Q = \Delta E_{int} + W$ $\therefore$ $Q = \Delta E_{int}$ $\qquad E_{int} = \frac{3}{2} nRT$

$\therefore \Delta E_{int} = \frac{3}{2} nR(T_B - T_A) = \frac{3}{2}(P_B V_B - P_A V_A)$

$= \frac{3}{2}\left((2\times10^5\text{Pa})(10^{-2}\text{m}^3) - (10^5\text{Pa})(10^{-2}\text{m}^3)\right)$

$= +1500\text{ J}$ $\qquad \boxed{Q = 1500\text{ J}}$

positive : heat $\boxed{\text{added}}$

b) $W = \int P dV = nRT_B \int \dfrac{dV}{V}$

$= nRT_B \ln\left(\dfrac{V_C}{V_B}\right)$

$= P_B V_B \ln\left(\dfrac{30\,\ell}{10\,\ell}\right) = (2\times10^5\text{Pa})(10^{-2}\text{m}^3)\ln 3 = \boxed{+2197\text{ J}}$

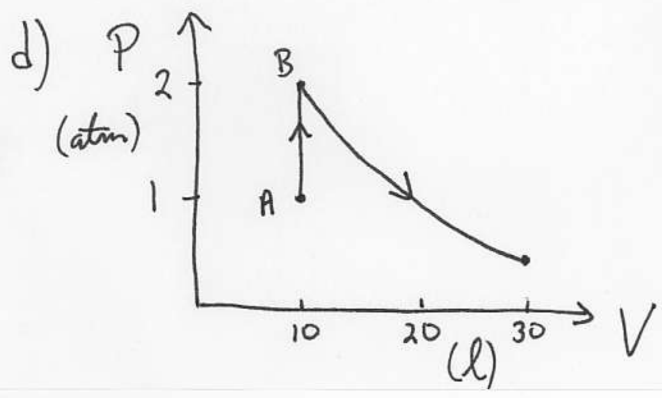
positive : work done $\boxed{\text{by gas}}$

c) $Q = \Delta E_{int} + W$; isothermal $\therefore \Delta E_{int} = 0$

$\therefore Q = W \qquad Q = \boxed{+2197\text{ J}}$

positive : $\boxed{\text{heat added}}$

d)



Problem 2.

A nuclear power plant in Virginia produces 1 Gigawatt (10^9 W) of electrical power. The plant operates between a high temperature of 380 °C and dumps heat into the James river at a lower temperature of 27 °C. The plant operates at an efficiency which is 2/3 of the maximum (Carnot) efficiency for these temperatures.

a) What is the rate at which energy is deposited into the river?

b) The Enviromental Protection Agency limits the maximum allowed temperature increase of the river water to be 10°C. What flow (in kg/s) of river water is required to assure that this limit is not exceeded?

a) want Q_L (heat into low-temperature reservoir)

$$T_H = 380°C = 653\text{ K} \qquad T_L = 27°C = 300\text{ K}$$

$$\varepsilon_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{K}}{653\text{K}} = 0.541$$

$$\varepsilon = \frac{2}{3}\varepsilon_{Carnot} = 0.360$$

Consider heat, work done in 1 sec. Power $= \frac{\text{Energy}}{\text{time}}$ $\therefore$ 10^9 J work each second

$$\varepsilon = \frac{W}{Q_H} \quad \therefore \quad Q_H = \frac{W}{\varepsilon} = \frac{10^9\text{ J}}{0.360} = 2.78\times10^9\text{ J}$$

$$Q_L = Q_H - W = 1.78\times10^9\text{ J} \quad \text{(each second)}$$

$$\therefore \text{ power into river} = 1.78\times10^9 \frac{\text{J}}{\text{s}} = \boxed{1.78\text{ GW}}$$

b) River temperature increases due to Q_L ...

$$Q_L = mc\Delta T$$

$$\therefore \frac{dQ_L}{dt} = \frac{dm}{dt}c\Delta T = \text{power into river}$$

$$\therefore \frac{dm}{dt} = \frac{dQ_L/dt}{c\Delta T} = \frac{1.78\times10^9\text{ J/s}}{\left(4186\frac{\text{J}}{\text{Kg}\cdot\text{K}}\right)(10\text{ K})} = \boxed{4.25\times10^4\text{ Kg/s}}$$

(42 metric tons per second)

Problem 3.

A 50 g ice cube, initially at -30°C, is dropped into an insulated flask which contains 1.35 kg of ethyl alcohol, initially at 70 °C. The ice is observed to melt completely. Ignore any heat flow into or from the flask.

a) What is the final temperature of the liquid mixture? Some possibly useful properties: latent heat of fusion for water = 3.33×10^5 J/kg, latent heat of vaporization of water = 2.26×10^6 J/kg, specific heat of liquid water = 4186 J/kg·K, specific heat of ice = 2100 J/kg·K, specific heat of liquid ethyl alcohol = 2400 J/kg·K.

b) By how much does the entropy of the ethyl alcohol decrease in this process?

c) Explain, in one or two sentences, why this decrease in entropy does not violate the Second Law of Thermodynamics.

a) Calorimetry : $\Sigma Q = 0$

$m_i = 50\times10^{-3}$ Kg $m_a = 1.35$ Kg $L_f = 3.33\times10^5$ J/Kg $C_i = 2100 \frac{J}{Kg\cdot K}$ $C_w = 4186 \frac{J}{Kg\cdot K}$

$C_a = 2400 \frac{J}{Kg\cdot K}$

$$\Sigma Q = \underbrace{m_i C_i (0^\circ - (-30^\circ C))}_{\text{ice warms to melting point}} + \underbrace{m_i L_f}_{\text{ice melts}} + \underbrace{m_i C_w (T - 0^\circ C)}_{\text{melted ice warms to } T} + \underbrace{m_a C_a (T - 70^\circ C)}_{\text{alcohol cools to } T} = 0$$

solve for T :

$$T(m_i C_w + m_a C_a) = m_a C_a (70^\circ C) - m_i C_i (30^\circ C) - m_i L_f$$

$$T = \frac{m_a C_a (70^\circ C) - m_i C_i (30^\circ C) - m_i L_f}{m_i C_w + m_a C_a} = \boxed{60^\circ C}$$

$= 333$ K

b) $$\Delta S_a = \int \frac{dQ}{T_a} = \int_{343K}^{333K} \frac{m_a C_a \, dT_a}{T_a} = m_a C_a \ln\left(\frac{333K}{343K}\right) = \boxed{-95.9 \text{ J/K}}$$

or, since ΔT_a is small use approximation $\Delta S_a \simeq \frac{Q}{T_{avg}} = \frac{m_a C_a \Delta T}{338K}$

$= -95.9$ J/K

(same to 3 figures)

c) The ethyl alcohol is not a closed system ∴ its entropy can decrease. The entropy increase of the ice (melted into water) is larger than the entropy decrease of the alcohol so we still have $\Delta S_{closed} \geq 0$ (one version of the Second Law).

Problem 4.

a) What is the root-mean-square speed of oxygen molecules contained in a volume of 10^4 litres at 2.0 atm if the total amount of oxygen is 2000 mol? Oxygen is a diatomic molecule with a molecular mass of 32 g/mol.

b) How many molecules are in one cubic centimeter?

a) $v_{rms} = \sqrt{\frac{3KT}{m}}$ need T, m

$$m = \frac{32\times10^{-3}\,Kg}{mol} \times \frac{1\,mol}{6.022\times10^{23}} = 5.31\times10^{-26}\,Kg$$

$$PV = nRT \quad \therefore\ T = \frac{PV}{nR} = \frac{(2\times10^5\,Pa)(10\,m^3)}{(2000\,mol)(8.315\,\frac{J}{mol\cdot K})} = 120\ K$$

$$\therefore\ v_{rms} = \sqrt{\frac{3(1.38\times10^{-23}\,\frac{J}{K})(120\,K)}{5.31\times10^{-26}\,Kg}} = \boxed{306\ m/s}$$

b) $$\frac{2000\,mol}{10^4\,l} = \frac{2000\,mol}{10^7\,ml}$$ $1\,ml = 1\,cc\ \ (1\,(cm)^3)$

$$= 2\times10^{-4}\,mol/cc$$

$$2\times10^{-4}\,\frac{mol}{cc} \times 6.022\times10^{23}\,\frac{molecules}{mol} = \boxed{1.20\times10^{20}\ \text{per cc}}$$